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Directed type theory and weak factorization systems

In this talk, I will propose a directed type theory. The goal of this project is to develop a type theory with semantics in both category theory and directed homotopy theory [1, 2]. Previous work (e.g. [3, 4, 5, 6]) has focused on developing syntax for the former semantics, but our work is motivated also by the latter.

At the core of this type theory is a ‘homomorphism’ type former whose terms are meant to represent homomorphisms or directed paths. Its rules are roughly analogous to those of Martin-Löf’s identity type. However, while the terms of an identity type can be reversed (so that from an identity $a = b$, one can obtain an identity $b = a$), the terms of the proposed homomorphism type cannot necessarily be reversed. Thus, while the homotopy theory that an interpretation of Martin-Löf’s identity type describes is in some sense symmetric, the homotopy theory that an interpretation of this proposed homomorphism type describes is asymmetric or *directed*.

I will give an interpretation of this type former into a standard weak factorization system (cofibrantly generated by the inclusion of a domain into a morphism) in the category of small categories. I will then describe constructions of more general directed weak factorization systems, and examples in categories of directed topological spaces and connected cubical sets.

REFERENCES:

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- [5] E. Riehl and M. Shulman, “A type theory for synthetic ∞ -categories”, *Higher Structures* 1 (2017) 116–193.
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