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Categorified cyclic operads in nature

In this talk, we shall introduce the notion of categorified cyclic operad, by focusing on their place and use "in nature". Categorified cyclic operads are like symmetric monoidal categories, in that they guide an interplay of commutativity and associativity, but they are more restrictive, as they allow less instances of these two isomorphisms. In particular, the coherence conditions of symmetric monoidal categories do not ensure coherence of categorified cyclic operads, the hexagon of Mac Lane not even being well-defined in the latter setting. The coherence conditions that we do take from Mac Lane are the pentagon and the requirement that the commutator isomorphism is involutive, but we need much more in order to ensure coherence: we need two more mixed coherence conditions (i.e. coherence conditions that involve both associator and commutator), a hexagon (which is *not* the hexagon of Mac Lane) and a decagon, as well as three more conditions which deal with the action of the symmetric group.

We first give an example of a categorified cyclic operad in the form of an easy generalisation of the structure of profunctors of Bénabou [1]. Essentially, profunctors admit the structure of a categorified cyclic operad because the cartesian product of sets (figuring in the definition of the composition of profunctors) is neither associative nor commutative on the nose.

We then show how to exploit the coherence conditions of categorified cyclic operads in proving that the Feynman category for cyclic operads, introduced by Kaufmann and Ward in [3], admits an odd version, which is, in turn, precisely the Feynman category for anticyclic operads.

We finish with combinatorial aspects of categorified cyclic operads, i.e. with their possible characterisations in convex and discrete geometry. This investigation, which is currently in progress, aims at finding polytopes which describe the coherences of categorified cyclic operads, in the same was as the geometry of symmetric monoidal categories is demonstrated by permutoassociahedra, or the geometry of categorified operads by hypergraph polytopes [2]. By changing the set of canonical isomorphisms of categorified cyclic operads, a 3-dimensional convex polytope, which contains six *enneagonal* facets and looks like a truncated associahedron, arises.

References:

[1] J. Bénabou, Les distributeurs, *Université Catholique de Louvain, Institut de Mathématique Pure et Appliquée*, rapport 33 (1973)

[2] K. Došen, Z. Petrić, Hypergraph polytopes, *Topology and its Applications*, 158 (2011), 1405–1444.

[3] R. M. Kaufmann, B. C. Ward, Feynman categories, *Astérisque (Société Mathématique de France)*, Numéro 387 (2017) Vol. 32, No. 12 (2017) 396–436.

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