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A categorical explanation of why Church's Thesis holds in the Effective Topos

Hyland's Effective Topos $\mathcal{E}\!f\!f$, introduced in [4], is one of the most remarkable examples of the Tripos-To-Topos construction (TTT) of [3]. In the topos the combined use of realizability and category theory has been extensively employed to analyze a variety of aspects of constructive mathematics. Beside TTT, it is well know that $\mathcal{E}\!f\!f$ can be obtained at least in two other ways: as the exact completion of the regular category $\mathcal{A}sm$ of assemblies as in [1, 2], and as the exact completion of the finite limit category $\mathcal{PA}sm$ of partitioned assemblies as in [10]. Similarly to the TTT, these two constructions can be described in terms of universal constructions of particular doctrines [8, 1]. In particular, the exact completion of a finite limit category is an instance of the more general *elementary quotient completion* (EQC), see [7, 6].

One of the benefits to see a category as the result of a multiplicity of different constructions is that one can prove properties of the category in the most convenient setting. In this talk we employ the characterization of $\mathcal{E}\!f\!f$ and of $\mathcal{A}sm$ as EQC (presented at CT17), to explain directly aspects of the topos which are not immediately visible when one looks at it as a TTT. The key point is that the EQC preserves (and reflects) all the properties that we consider, *e.g.* it preserves a natural number object in the base category, whereas TTT does not.

Two aspects of $\mathcal{E}\!f\!f$ that we will consider, relevant from the perspective of constructive mathematics, are that the internal logic of $\mathcal{E}\!f\!f$ extends Kleene's realizability interpretation of Intuitionistic Arithmetic as in [5] and satisfies Church's Thesis. Neither property holds for the original tripos that produces $\mathcal{E}\!f\!f$ via the TTT, while both are direct consequences of considering $\mathcal{E}\!f\!f$ and $\mathcal{A}sm$ as EQC.

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