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Conjugation semigroups and conjugation monoids with cancellation

In this talk we introduce the category of conjugation semigroups which can be seen as an abstraction of conjugation of complex numbers or of quaternions. A conjugation semigroup

$$(S, +, \overline{(\ )})$$

is a, not necessarily commutative, semigroup $(S, +)$ equipped with a unary operation $\overline{(\ )} \colon S \to S$ satisfying the following identities: $x + \overline{x} = \overline{x} + x$, $x + \overline{y} + y = y + \overline{y} + x$, and $\overline{x + y} = \overline{y} + \overline{x}$. The quasivariety of conjugation semigroups with cancellation is a weakly Mal'tsev category and we present a characterization of all admissible diagrams in the sense of [2].

In the subcategory of conjugation monoids with cancellation we describe, for Schreier split epimorphisms with codomain B and kernel X

$$X \underset{k}{\overset{q_f}{\rightleftarrows}} A \underset{f}{\overset{r}{\rightleftarrows}} B \ ,$$

all morphisms $h \colon X \to B$ which induce a reflexive graph, an internal category or an internal groupoid. That is, all morphisms $h \colon X \to B$ that induce a morphism $\tilde{h} \colon A \to B$, with $\tilde{h}k = h$, such that

$$A \overset{f}{\underset{\tilde{h}}{\overset{\longleftarrow r \longrightarrow}{\rightrightarrows}}} B$$

gives rise to a reflexive graph, an internal category or an internal groupoid.

We show that a relative version of the so-called "Smith is Huq" condition holds for Schreier split epimorphisms in the category of conjugation monoids with cancellation as well as other relative conditions.

REFERENCES:

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