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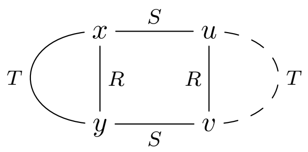
Observations on the Shifting Lemma

Given a variety of universal algebras, the **Shifting Lemma** [2] states: given congruences R, S and T on the same algebra X such that $R \wedge S \leqslant T$, whenever x, y, u, v are elements in X with $(x, y) \in R \wedge T$, $(x, u) \in S$, $(y, v) \in S$ and $(u, v) \in R$, it then follows that $(u, v) \in T$



A variety of universal algebras satisfies the Shifting Lemma precisely when it is congruence modular [2]. In a regular categorical context it is easy to see that congruence modularity implies that the categorical formulation of the Shifting Lemma holds. Since regular Mal'tsev or (regular) Goursat categories are such that their lattices of equivalence relations on any object are modular [1], then the Shifting Lemma holds in both contexts.

In this talk we prove that regular Mal'tsev categories and Goursat categories may be characterised through stronger variations of the Shifting Lemma. More precisely, a regular category $\mathcal{C}$ is a Mal'tsev category if and only if the Shifting Lemma holds for reflexive relations R, S, and T in $\mathcal{C}$. And a regular category $\mathcal{C}$ is a Goursat category if and only if the Shifting Lemma holds for a reflexive relation S and reflexive and positive relations R and T in $\mathcal{C}$ (a positive relation is of the form $U^{\circ}U$, for some relation U).

References:

[1] A. Carboni, G.M. Kelly, M.C. Pedicchio, *Some remarks on Maltsev and Goursat categories*, Appl. Categ. Structures 1, no.4 (1993) 385–421.

[2] H.P. Gumm, *Geometrical methods in congruence modular algebras*, Mem. Amer. Math. Soc. 45 (1983), no. 286.

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