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Skew monoidal structure on categories of algebras

There is a long tradition of constructing monoidal or closed structure on the category of algebras for a monad that is assumed to be commutative, monoidal, cartesian closed, or similar. In each case, one builds a tensor product classifying bilinear maps [6] using a coequalizer. This approach, initiated by Linton's description of the construction [8], has been studied by Kock, Guitart and others [7, 4, 1, 5], while Seal has recently examined the monoidal case in some detail [9]. In this talk we explore these ideas for skew monoidal categories, *viz.* suitably directed versions of monoidal categories in which the structural maps are not assumed to be invertible [10]. Under standard conditions I will show that, for a strong monad T on a skew monoidal category, the category of T -algebras acquires a skew monoidal structure with a tensor product classifying left-linear maps. I will then characterise the monoids for this left-linear monoidal structure as precisely the T -monoids of Fiore *et al.* [2, 3], and give two constructions for free monoids in skew monoidal categories.

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