Stone Representation Theorem represents Boolean algebras as subalgebras of power-set Boolean algebras. Banaschewski and Bhutani ([1]) and Borceux et al. ([2]) have introduced Stone representations for Boolean algebras in a topos of sheaves on a locale. It is desirable to have such a construction in other topoi.

The presentation considers Stone representation for Boolean algebras in the topos \( \text{MSet} \) of \( M \)-sets for a monoid \( M \), and its subtopoi \( \text{Sh}_{j} \text{MSet} \), of \( j \)-sheaves determined by right ideals of \( M \). In order to obtain a suitable definition of a Stone map in these topoi, we need to know the counterpart of the Boolean algebra \( 2 \) (the initial Boolean algebra object) in them. Moreover, in order to internalize the power-set Boolean algebra \( (2^X, \text{for } X = \text{Hom}_{\text{Boo}}(A,2) \text{ in } \text{Set}) \) in our topos, we take the exponential object, and apply the notion of internal homomorphism introduced by Ebrahimi (in [3]) for algebras in any Grothendieck topos. Unlike the case for Boolean algebras in \( \text{Set} \), the Stone representation we introduce in \( \text{MSet} \) and in \( \text{Sh}_{j} \text{MSet} \) is not always a monomorphism; to be so we will find necessary and/or sufficient conditions on \( M \). For instance, we will see that for a finite monoid \( M \), the Stone representation is a monomorphism if and only if \( M \) is a group.

References:


*Joint work with Mojgan Mahmoudi.