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Stone Representation Theorem for Boolean algebras in the topos of (pre)sheaves on a monoid

Stone Representation Theorem represents Boolean algebras as subalgebras of power-set Boolean algebras. Banaschewski and Bhutani ([1]) and Borceux et al. ([2]) have introduced Stone representations for Boolean algebras in a topos of sheaves on a locale. It is desirable to have such a construction in other topoi.

The presentation considers Stone representation for Boolean algebras in the topos **MSet** of M-sets for a monoid M, and its subtopoi $\mathcal{S}h_{j^I}\mathbf{MSet}$, of j^I-sheaves determined by right ideals of M. In order to obtain a suitable definition of a Stone map in these topoi, we need to know the counterpart of the Boolean algebra **2** (the initial Boolean algebra object) in them. Moreover, in order to internalize the power-set Boolean algebra ($\mathbf{2}^X$, for $X = Hom_{Boo}(A, \mathbf{2})$ in **Set**) in our topoi, we take the exponential object, and apply the notion of internal homomorphism introduced by Ebrahimi (in [3]) for algebras in any Grothendieck topos. Unlike the case for Boolean algebras in **Set**, the Stone representation we introduce in **MSet** and in $\mathcal{S}h_{j^I}\mathbf{MSet}$ is not always a monomorphism; to be so we will find necessary and/or sufficient conditions on M. For instance, we will see that for a finite monoid M, the Stone representation is a monomorphism if and only if M is a group.

References:

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