Lurdes Sousa
IPV, Viseu & CMUC, Coimbra

Kan-injectivity and KZ-monads

Full reflective subcategories are just Eilenberg-Moore categories of idempotent monads. Freyd and Kelly [5] formulated and gave a positive answer to the Orthogonal Subcategory Problem, asking whether an orthogonal subcategory is reflective. In a category $\mathcal{X}$, an object A is said to be orthogonal to a morphism h provided that $\mathcal{X}(A, h)$ is an isomorphism. In the setting of 2-categories, if instead of an isomorphism we have a right adjoint retraction, we obtain the notion of (left) Kan-injectivity of A with respect to h. This notion is related to KZ-monads (or lax idempotent monads) [7]. In order-enriched categories, the Eilenberg-Moore category of a KZ-monad is a (locally full, in general non full) subcategory of the base category: it is called a KZ-monadic subcategory. Based on the papers [1, 2, 3, 4, 6, 8], I will show that, in several aspects, the richness of the interplay between orthogonality and full reflective subcategories still remains for Kan-injectivity and KZ-monadic subcategories. A particular attention will be given to examples concerning locales and topological spaces.

References:

[1] J. Adámek, L. Sousa, KZ-monadic categories and their logic, *Theory Appl. Categ.* 32 (2017) 338–379.

[2] J. Adámek, L. Sousa, J. Velebil, Kan injectivity in order-enriched categories, *Math. Structures Comput. Sci.* 25 (2015) 6–45.

[3] M. Carvalho, L. Sousa, Order-preserving reflectors and injectivity, *Topology Appl.* 158 (2011) 2408–2422.

[4] M. Carvalho, L. Sousa, On Kan-injectivity of locales and spaces, *Appl. Categ. Structures* 25 (2017) 83–104.

[5] P. J. Freyd, G. M. Kelly, Categories of continuous functors I, *J. Pure Appl. Algebra* 2 (1972), 169–191.

[6] D. Hofmann, L. Sousa, Aspects of algebraic algebras, *Log. Methods Comput. Sci.* 13 (2017).

[7] A. Kock, Monads for which structures are adjoint to units, *J. Pure Appl. Algebra* 104 (1995) 41–59.

[8] L. Sousa, A calculus of lax fractions, *J. Pure Appl. Algebra* 221 (2017) 422–448.