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Algebraic structure from non algebraic proofs

In a model structure (trivial) cofibrations, (trivial) fibrations, and weak equivalences are classes of maps that satisfy certain axioms. Meanwhile, in an algebraic model structure, as defined by Riehl [2], (trivial) cofibrations are objects in a category of coalgebras over a comonad and (trivial) fibrations belong to category of algebras over a monad, while weak equivalences are still just a class of maps. Motivated by a construction of identity types in cubical sets, I introduced an even more structured notion where weak equivalences also have structure. In place of the 3-for-2 axiom, one has a *functorial 3-for-2 operator*, which assigns weak equivalence structures to maps, while respecting homomorphisms of weak equivalences.

In [3] Sattler gave general criteria for the existence of model structures, which gives a constructive proof that Bezem-Coquand-Huber cubical sets admit a model structure, when combined with Huber's result [1] that cubical sets contain a fibrant universe. To obtain a functorial 3-for-2 operator from Sattler's proof, it seems at first that one would need to carry out a tedious process of analysing each step of the proof, changing it to a "functorial" version and then checking the proof generalises.

The subject of this talk is a much easier construction of the functorial 3-for-2 operator for cubical sets, that mostly avoids tedious bookkeeping. The key idea is that assuming the existence of (constructive) inaccessible sets, instead of working over the category of cubical sets itself, we can work over the Grothendieck fibration of (large) category indexed families. We apply Sattler's result to show that each fibre of the fibration is a model structure in the usual (non-algebraic) sense, and then use this to derive a functorial 3-for-2 operator and other structure on the original category. Even arguments based on cofibrantly generated awfs's can carried out in the fibration by using the notions of fibred lifting problem and universal lifting problem from [4].

This technique promises to be, in some respects, very general. In particular, I expect it to apply to cubical assemblies, where the underlying category is not cocomplete, and the most appropriate notion of lifting problem is not Garner's algebraic lifting problem, but instead either internal lifting problems or computable algebraic lifting problem (these appear as lifting problems over codomain fibrations and lifting problems over internal category indexed families of presheaves respectively in [4]).

References:

[1] S. Huber. *A Model of Type Theory in Cubical Sets.* Lic. thesis, Univ. Gothenburg, 2015.

[2] E. Riehl. Algebraic model structures. *New York J. Math.*, 17:173-231, 2011.

[3] C. Sattler. The equivalence extension property and model structures. arXiv:1704.06911 (2017).

[4] A.W. Swan. Lifting problems in Grothendieck fibrations. arXiv:1802.06718 (2018).