On cartesian functors

Jean Bénabou

Let $P : \mathcal{C} \to \mathcal{B}$, $P' : \mathcal{C}' \to \mathcal{B}$ and $F : \mathcal{C} \to \mathcal{C}'$ be functors such that $P'F = P$. If P and P' are fibrations, F is cartesian if it preserves cartesian maps.

However if P and P' are not fibrations, there may be very few cartesian maps and the preservation of such maps will be a very weak property of F from which not many results will follow.

We propose a definition of cartesian functors for P and P' arbitrary, which of course reduces to the usual one when they are fibrations, but which applies to a much wider range of mathematically significant examples.

We show that most of the results of the "classical" theory of cartesian functors between fibered categories can be proved in this general setting.

Moreover, the proof of some of these results under much weaker assumptions forces to disclose some adjunction properties which were not known, even in the case of fibrations.