

Descent theory for span enriched categories

Renato Betti

We deal with descent theory from the point of view of *Span*-enriched categories. These comprise in particular locally internal categories and existential hyperdoctrines in the sense of Lawvere.

The point of view of enriched categories allows a common setting for internal and locally internal categories, thus simplifying many proofs, and we believe that it gives intuitive insights. Moreover, it provides a natural meaning to the diagrams which arise when dealing with such categories. Among other things, this allows to give to the whole subject a certain formal analogy with the original formulation of descent theory as in Grothendieck.

If \mathcal{C} is a finitely complete category with stable coequalizers and \mathcal{X} is a locally internal category over \mathcal{C} , any universal regular epimorphism is an effective \mathcal{X} -descent if and only if \mathcal{X} is “*p-cocomplete*”. This means that suitable right Kan extensions along p exist and are representable in the bicategory of $(Span\ \mathcal{C})$ -enriched categories. As a consequence, universal regular epimorphisms in \mathcal{C} are effective \mathcal{X} -descent morphisms for any cocomplete, locally internal category \mathcal{X} over \mathcal{C} . If \mathcal{X} is the “principal” category over the locally cartesian closed category \mathcal{C} , i.e. \mathcal{C} regarded as locally internal over itself, one recovers the original case.