A modal logic for localic local maps of toposes

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In this talk we report on part of the research on the Logic of Types and Computation at Carnegie Mellon University [1], as outlined in the talk by Steven Awodey.

Using the axiomatic setup for local maps of toposes explained in Awodey's talk, we present a modal logic for localic local maps of toposes.

To make the ideas precise we make use of the notion of a tripos, which has been used to successfully describe many different (realizability and other) toposes. A tripos is a categorical notion of model for intuitionistic higher-order logic without extensionality for propositions, and there is a well-known way of taking any tripos to the topos represented by the tripos.

We define the concept of a "local" tripos, which is a tripos with a modality operator $\sharp$ on propositions. We show how any localic local map of toposes gives rise to a local tripos, and how any local tripos gives rise to a localic local map of toposes.

As a concrete example, we show how any partial combinatory algebra A and a sub-pca $A_\sharp \subseteq A$ gives rise to a local tripos.

REFERENCES

[1] D.S. Scott et. al., *Logics of Types and Computation at Carnegie Mellon University*, http://www.cs.cmu.edu/Groups/LTC/.