Baer sums and fibered aspects of Mal'cev operations

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The geometrical meaning of certain axioms for Mal'cev operations is emphasized and from that its classical associated action, when the operation is associative, can be derived in the general context of the Barr exact categories. This action is defined via a direction functor d which is shown to be a cofibration preserving the products and the terminal object. We are then in the situation where any group structure on an object X in the basis of d induces canonically a monoidal structure on the fibre above X. As an illustration, the Baer construction of the sum of two group extensions with abelian kernel as well as the associated group structure on them are immediately restored. On the other hand, this same cofibration d will allow us to establish a relationship between the notions of naturally Mal'cev and essentially affine category.