Quantaloids describing propagation of physical properties

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The base of this talk lies in the physical duality between the state space and property lattice representations of a physical system (Piron 1976), its formulation as a categorical equivalence (Moore 1995, 1997, 1999) and its extension to a duality of quantaloids of so called 'operational resolutions' (Amira *et al.* 1998, Coecke and Stubbe 1999a, 1999b). In particular, since the lattice meet in a property lattice is semantic conjunction (Piron 1977), temporal causation is a meet preserving map $f^* : \mathcal{L}_2 \to \mathcal{L}_1$ between property lattices, with $a_1 = f^*(a_2)$ being the cause of a_2 in $\mathcal{L}_1$, that is, the weakest property in $\mathcal{L}_1$ whose actuality guarantees the actuality of a_2 in $\mathcal{L}_2$ (Faure *et al.* 1995, Coecke 1999, Moore 1999). Its join preserving Galois dual is then a map describing the propagation of 'certainly true properties'. Exploiting this fact leads to a duality of quantaloids, relating on the one hand these conjunction preserving maps, encoding temporal causation, and on the other the join preserving maps, encoding propagation of properties. On another level, the existing duality between the state space description and the property lattice description of a system, formulated here as an 'operational resolution', leads to the consideration of the propagation of states and 'actuality sets', i.e., collections of properties in which at least one property is actual. Again, a duality of quantaloids emerges naturally, this time relating property propagation with state propagation. Ultimately, this construction, exhibiting the subtle relation between 'possible state propagation' and 'certain property propagation', shows the advantage of categorical methods in the study of non-classical physical systems.

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