

Higher-dimensional Mac Lane's pentagon and Zamolodchikov equations

Sjoerd Crans

Mac Lane's pentagon is an equality between two composites of associativity constraints, and arises from composing four 1-arrows in a bicategory in different ways. A higher-dimensional version of this pentagon lives in somewhat weak 6-dimensional teisi, where 'horizontal' composition is dimension raising, and in which there are (6-dimensional) associativity constraints for horizontal composition of 2-arrows. The higher-dimensional pentagon arises from composing four 2-arrows in all possible ways.

Looping twice, the (5-dimensional) 'vertices' of the higher-dimensional pentagon are proofs of the Zamolodchikov equation in a braided monoidal 2-category. Hence the higher-dimensional pentagon expresses the relations between these proofs concisely.