

Kan extensions of internal functors. Algebraic approach

Tamar Datuashvili

We consider the notion of the Kan extension for internal functors in the category Groups of groups. The internal nature of categories enables us to consider them as crossed modules [6] and to think on the problem of necessary and sufficient conditions for the existence of internal Kan extensions suggested by G. Janelidze. Thus we follow an algebraic approach to this problem and use homological algebra methods as in [1]-[3] to study various categorical notions and relations between them. Under certain conditions we establish the necessary and sufficient conditions for the existence of internal Kan extension. Questions related to this problem are also discussed [4]. Due to Mac Lane - Whitehead's well-known classification of connected cell complexes according to their 3-type [5] we can also consider the topological approach to this question; it will be treated in a forthcoming paper. Since every internal category in Groups is a groupoid, this kind of questions can be treated by means of the category theory (groupoid) methods. Note that in our case the groupoid approach did not give the desirable result.

REFERENCES

- [1] T. Datuashvili, *Cohomology of internal categories in categories of groups with operations*, Proc. Conf. Categorical Topology, Prague 1988 (World Scientific 1989), 270-283.
- [2] T. Datuashvili, *Cohomologically trivial internal categories in categories of groups with operations*, Applied Categorical Structures 3 (1995), 221-237.
- [3] T. Datuashvili, *Whitehead homotopy equivalence and internal category equivalence of crossed modules in categories of groups with operations*, Proc. A. Razmadze Math. Inst. 113 (1995), 3-30.
- [4] T. Datuashvili, *Kan extensions of internal functors. Algebraic approach*, Georgian Mathematical Journal 6 (1999), 127-148.
- [5] S. Mac Lane and J.H.C. Whitehead, *On the 3-type of a complex*, Proc. National Academy of Sciences 36 (1950), 41-48.
- [6] T. Porter, *Extensions, crossed modules and internal categories in categories of groups with operations*, Proc. Edinburgh Math. Soc. 30 (1987), 373-381.