

What is a double category like?

Robert Dawson

Double categories were introduced by Ehresmann in the 1960's. A double category can be thought of as a collection of “cells” that form a category under two composition operations that obey the *middle-four exchange* axiom $(\alpha \circ \beta) \bullet (\gamma \circ \delta) = (\alpha \bullet \gamma) \circ (\beta \bullet \delta)$ whenever both sides are defined. This paper presents joint work with R. Paré in which we examine the relationships between double categories and geometry, topology, computability, and combinatorics.

We shall show, in particular, that the visual metaphor by which we represent a cell as a rectangle in the plane has deeper significance than might be expected. We will see that this can be generalized, but not perfectly, to higher dimensions. This has implications for the use of diagrams in higher-dimensional category theory, as it makes more exact the extent to which such diagrams can be considered as reliable depictions of multiple categories.

We will also see that various combinatorial and computational structures can be represented as hom-sets in free double categories.