

On small-orthogonality classes in locally presentable categories

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A full subcategory \mathcal{K} of a locally λ -presentable category \mathcal{L} is called λ -orthogonal in \mathcal{L} if there exists a set of morphisms M between λ -presentable objects of \mathcal{L} such that the \mathcal{K} -objects are precisely the \mathcal{L} -objects which are orthogonal to M . It is well-known that \mathcal{K} is then reflective in \mathcal{L} , actually in a strong sense, and explicit constructions of the reflection morphisms have been given. We present a new and simpler such construction, which allows to see some non-obvious properties of λ -orthogonality classes. In categorical terms, \mathcal{K} is λ -orthogonal in \mathcal{L} precisely when the corresponding theory morphism is a quotient morphism. In syntactic terms, we have the following: seeing \mathcal{L} as the category of models of a uniform λ -ary limit-theory \mathcal{T} , λ -orthogonality clearly means that \mathcal{K} can be axiomatized by adding new (uniform λ -ary) limit sentences to \mathcal{T} (in the same language). The construction allows us to see easily that related properties, which seem a priori stronger, actually hold.

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