On the non-compactness of realcompact spaces

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Categorical closure operators have been used to develop a comprehensive theory of compactness in a general category (cf.[1]). For a closure operator c on a category $\mathcal{X}$, $X \in Ob\mathcal{X}$ is c-compact iff the projection $X \times Y \xrightarrow{\pi_Y} Y$ is c-closed for any $Y \in Ob\mathcal{X}$.

We first observe that there is *no* closure operator c on the category of topological spaces (and continuous maps) for which the realcompact spaces are exactly the c-compact objects. There is however a closure operator which describes these spaces as absolutely closed.

This consideration leads to a new definition; that of *ck-closed* morphism in a category, and the associated notion of ck-compactness. (Both c and k are closure operators on the category.) A theory analogous to that of compactness is developed, which – amongst other things – extends the Lindelöf property to a general setting.

References

[1] M.M.Clementino, E.Giuli, W.Tholen. Topology in a category: Compactness. *Portugaliae Math.*, **53** (1996) 397-433.