Relational (co)algebras for the power-set monad

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As is well known, the covariant power-set functor on the category **Set** of sets and functions carries a monad structure, whose algebras are the complete join-semilattices. It was recently observed by Martin Hyland and Andrea Schalk that the power-set functor has a natural extension to an endofunctor of the category **Rel** of sets and relations, and that its monad structure also extends; the algebras for this monad (which may also be thought of as coalgebras, since **Rel** is self-dual) are potentially of interest as models of non-deterministic transition systems. We give a description of these algebras as ordered sets: they include all posets which are multicocomplete in the sense of Diers, and more besides. Noting that **Rel** is the Kleisli category of the power-set monad on **Set**, we also ask what it means for a monad on a general category to admit an extension to its own Kleisli category: this appears to be a strong commutativity condition on the monad.