Categories with sums and right distributive tensor product

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Models for parallel and concurrent processes lead quite naturally to the study of monoidal categories [1]. In particular a category **Tree** of trees equipped with a non symmetric tensor product, interpreted as concatenation, seems to be of particular interest in representing (local) behaviour of non deterministic agents able to communicate [2]. This category is also provided with a coproduct (corresponding to choice between behaviours) and the tensor product is only partially distributive w.r.t. it, in order to preserve non determinism.

More specifically, $\mathbf{Tree(A)}$ is the full subcategory of $\mathcal{A}$-*SymCat* consisting of finite objects, where $\mathcal{A}$ is the 2-category induced by the free monoid generated from the set A. Because of its very nature $\mathbf{Tree(A)}$ has finite coproducts and initial object, but it inherits from the free monoid the concatenation operation as a non-symmetric monoidal functor, that distributes on the right w.r.t. coproducts. In this paper we prove that, given an alphabet A, $\mathbf{Tree(A)}$ is equivalent to the free category generated by A and enjoying the above mentioned properties. We provide a a deductive system and a list of equations defining such a category [4], as well as normalization theorems both for its objects and its morphisms intended as terms. By exploiting this normal forms, we prove the main result, providing in this way a concrete description of a free structure.

References

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