On the monadicity of categories with chosen colimits

Stephen Lack*

There is a 2-category \( \mathcal{J}\text{-Colim} \) of small categories equipped with a choice of colimit for each diagram whose domain \( J \) lies in a given small class of small categories, functors strictly preserving such colimits, and natural transformations. The evident forgetful 2-functor from \( \mathcal{J}\text{-Colim} \) to the 2-category \( \mathbf{Cat} \) of small categories is known to be monadic. We extend this result by considering not just conical colimits, but general weighted colimits; not just ordinary categories but enriched ones; and not just small classes of colimits but large ones; in this last case we are forced to move from the 2-category \( \mathcal{V}\text{-Cat} \) of small \( \mathcal{V} \)-categories to \( \mathcal{V} \)-categories living within some larger universe. In each case, the functors preserving the colimits in the usual “up-to-isomorphism” sense are recovered as the pseudomorphisms between algebras for the 2-monad in question.

*Joint work with G.M. Kelly.