Revisiting Eilenberg's structured objects*

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At one stage during his Columbia University lectures on homological algebra in the fall of 1962, Samuel Eilenberg presented an appealing approach, relatively unexplored since then, to the matter of objects in one category being endowed with structures coming from another.

Explicitly, given categories $\mathcal{C}$ and $\mathcal{D}$, and a set-valued functor $\Lambda : \mathcal{D} \to \mathcal{S}ets$, a Λ-*structure* on an object C of $\mathcal{C}$ was any functor $\gamma : \mathcal{C}^{op} \to \mathcal{D}$ for which $\Lambda \bullet \gamma$ and $\mathcal{C}(-, C) : \mathcal{C}^{op} \to \mathcal{S}ets$ were naturally equivalent, together with a choice of one such natural equivalence. For example, the abelian group objects in a category, by almost any definition, coincided, at least to within equivalence, with the objects endowed with U-structures, for $U : \mathcal{A}b \to \mathcal{S}ets$ the usual underlying-set functor on the category $\mathcal{A}b$ of abelian groups.

The exposition to be presented here will review this approach, along with some of its successes and failures, and explore its relation to alternative approaches to the problem of understanding the phenomenon of objects somehow residing simultaneously in two distinct categories.

*or *Eilenberg's notion of external structures on objects in categories, revisited.*