

Braided categories, moduli spaces of curves and the Grothendieck-Teichmüller group

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Following in part indications of A. Grothendieck in his ‘Esquisse d’un programme’, V. Drinfeld introduced the Grothendieck-Teichmüller group around 1990, in the pro-unipotent genus 0 case and in connection with deformations of braided categories. Since then the construction has been extended to the profinite case (by Y. Ihara) and especially to all higher genera, an extension which involves deep geometric properties of the moduli spaces of curves. In particular the role of MacLane’s coherence relations is played by the relations defining various complexes of curves, as introduced in a topological setting by W. Harvey, W. Thurston and others in the early eighties. We will review these recent advances, pointing the connection with fundamental groupoids of moduli spaces of curves and will try to outline the known and conjectural relationship with the absolute Galois group of the rationals.