Internal categorical (co)groups in topological spaces

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Given a pointed topological space (X, p), it is well known that the suspension $\Sigma(X, p)$ is an H-cogroup and the loop space $\Omega(Y, q)$ is an H-group. Thus $\pi_n(X, p) = [(S^n, *), (X, p)]$ is a group for $n \geq 1$ (even abelian for $n \geq 2$) and there is an action of $\pi_1(X, p)$ on $\pi_n(X, p)$, $n \geq 1$, giving $\pi_n(X, p)$, $n \geq 2$, the structure of $\pi_1(X, p)$-module.

In this work these results are raised to a categorical level which leads to higher homotopy structures. Using that Top_*, the category of pointed topological spaces, is enriched over the (monoidal) category *Gpd* of groupoids, we look for (braided or symmetric) categorical group structures in the track groupoid $Top_*((X, p), (Y, q))$. This lead us to define notions of internal categorical (co)group object in any enriched category over *Gpd*. Then we show the spaces $\Sigma(X, p)$ and $\Omega(Y, q)$ as important examples of such objects in Top_*, and define, for any $(X, p) \in Top_*$, higher homotopy groupoids $\rho_n(X, p) = Top_*((S^{n-1}, *), (X, p))$, $n \geq 2$. These groupoids carry a tensor structure in such a way that $\rho_2(X, p)$ turns out to be a categorical group whereas $\rho_n(X, p)$ is a braided categorical group if $n \geq 3$ and even a symmetric categorical group if $n \geq 4$. Also, with a suitable notion of internal action, we prove that $\rho_n(X, p)$, $n \geq 3$, turns out to be a $\rho_2(X, p)$-module.

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