

Chu Spaces and their Galois Connections

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We consider the functor Sl from the category Chu of Chu spaces and Chu transformations to the category $Slat$ of complete lattices and join preserving homomorphisms, which associates a Chu space $C = (X, A, \models)$ to the complete lattice $Sl(C)$ of the intersection closed family of subsets of X generated by the polar sets

$$a^* := \{ x \mid x \models a \} \quad a \in A.$$

We show that Sl is *-autonomous functor with an injection

$$Sl(C \multimap C') \rightarrow Slat(Sl(C), Sl(C')).$$

Even when $C \multimap C'$ is a trivial Chu space, the lattice $Slat(Sl(C), Sl(C'))$ can be nontrivial, which suggests the study of the $Slat$ -enriched category of Chu spaces with the hom-lattice $Slat(Sl(C), Sl(C'))$ might be useful.