Iterated dissolution locales

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A locale A has a Boolean coreflection if and only if the chain obtained by iterating the dissolution functor stabilizes. If it stabilizes then it does so at the Boolean coreflection of A.

A is called n-soluble if its n^{th} dissolution is Boolean. I give characterizations of 1-, 2-, 3- and 4-soluble locales, and an example of a 3-soluble, 2-insoluble space.

Supplementing these characterizations is a sufficient condition for complete insolubility. It implies that the space of rational numbers has no Boolean coreflection and hence that the only metrizable spaces with a Boolean coreflection are 1-soluble, i.e. scattered.