

# Ordinal sum, subdivision, joins and folding operations

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One of the possible models for weak infinity categories is that of weak Kan complexes. Although just a weakened form of a classical notion from algebraic topology, they can play the rôle of the nerves of weak infinity categories and have the advantage of coming with a pre-existing developed homotopy theory. As they form a subcategory of simplicial sets, we start by examining the basic category  $\Delta$  and the way certain parts of its structure are reflected in simplicial sets. This category  $\Delta$  comes with a simply defined pairing given by ordinal sum, that is concatenation of ordinals. Simple use of Kan extension techniques yields a subdivision functor for simplicial sets, distinct from barycentric subdivision. There is a related join functor and links with a codiagonal functor from bisimplicial objects to simplicial ones. Its use will be illustrated by the passage between two of the main models for homotopy 3-types (2-crossed modules and crossed squares) and the appearance of the subdivision diagram in the work by Brown and Gilbert on braided crossed modules. As a final application of ordinal sum, we will examine the folding operations used to define composition of homotopy coherent transformations.

This work builds on unpublished notes of Duskin and van Osdol and well known results on the  $\bar{W}$  functor but also reports on results from the theses of Ehlers and Mutlu and extensions of that work including some joint work with them.