Approximative spaces

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A *cover* of a poset $(P, \leq)$ is a subset $A \subseteq P$ such that

whenever $x \not\leq y$ there is an $a \in A$ and a $z \leq x, a$ such that $z \not\leq y$.

An *approximative space* is a poset $(P, \leq)$ together with a system of covers $\mathcal{A}$ satisfying a certain natural admissibility condition.

(The intuition behind the notion: The elements of P are approximations of some values (points), $x \leq y$ indicates that x is a better approximation then y ("the grain x is smaller than the grain y") and the individual covers represent precision requirements.)

An *approximate map* $(f, \varphi) : (P, \leq, \mathcal{A}) \to (\mathcal{Q}, \leq, \mathcal{B})$ consists of a map $\varphi : \mathcal{B} \to \mathcal{A}$ and a multivalued $f : P \to Q$ which restricts, for each $B \in \mathcal{B}$ to an $f_B : B \to \varphi(B)$ such that, roughly speaking, the (possibly many) values of an $f_B(b)$ can differ in the range of the given precision only.

A uniform (or, more generally, nearness) frame can be viewed as an approximative space, and a uniform localic morphism (relation inverse to a uniform frame homomorphism) can be viewed as an aproximative map. On the other hand, it can be shown that the category of approximative spaces and approximative maps can be densely extended to the category of complete nearness locales (densely in the sense that any aproximative space is "very close" to a complete nearness frame, and any approximative map is "very close" to a(n inverted) frame homomorphism.