Remarks on exponentiable morphisms in **Top**

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This is a report on work in progress. We try to describe the *exponentiable morphisms* in **Top**, i.e. $-\times p : \mathbf{Top}\ /_T \to \mathbf{Top}\ /_T$ is left adjoint, by elementary topological properties. In 1978/82, Susan B. Niefield characterised them very elegantly in terms of certain *binding* families $(\mathcal{H}_t)_T$ of *Scott-open* sets of open sets in the fibres X_t of p. Among others, she derived from that a nice elementary description of exponentiable embeddings. An embedding $p : X \hookrightarrow T$ is exponentiable in **Top** iff its image $p(X)$ is *locally closed* in T, i.e. it is the intersection of an open set with a closed one.

Meanwhile, there is a simple direct proof for the latter result. It implies exponentiability for all $p : X \to T$, X an exponentiable object in **Top**, i.e. *quasi-locally compact*, T *locally Hausdorff*, i.e. every point in T admits a neighborhood, that is a Hausdorff-space.

Arbitrary exponentiable morphisms $p : X \to T$ are characterized by the existence of a *saturated* neighborhood U_x $(= p^{-1}(U_x))$ for any $x \in X$ such that $p(U_x)$ is locally closed in T and the restriction $U_x \to p(U_x)$ of p is an exponentiable pullback-stable quotient in **Top**. This focusses our attention to the latter species. Moreover, it yields an elementary description of exponentiable monomorphisms in **Top**.