Some remarks on topological functors

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A theory of topological functors is presented in the book [1]. A topological functor is a faithful functor $U : A \to X$ that has unique $U$-initial lifts of $U$-structured sources. A theorem that R.-E. Hoffmann proved in his thesis [2] plays a very important role in our investigations. It says that from each category $A$ there is at most one topological functor into the category $\text{Set}$ of sets modulo natural isomorphy. In [3] he proves moreover that from each category $A$ there is at most one topological functor into a balanced category (i.e. a category in which each bimorphism is isomorphic) modulo equivalence. Functors $F : A \to X$ and $G : A \to Y$ are equivalent iff there exists an equivalence $H : X \to Y$ such that $H \circ F \cong G$.

We will see for functors $A \xrightarrow{U} B \xrightarrow{V} C$ if the composition $VU$ and $U$ are topological then so is $V$. We get that from each category $A$ there is at most one topological functor into a balanced basecategory modulo isomorphy, i.e. for topological functors $F : A \to X$ and $G : A \to Y$ into balanced categories there exists an isofunctor $H : X \to Y$ such that $H \circ F \cong G$. We will describe the basecategories $X$ into which there exists at most one topological functor from each category modulo natural isomorphy.

By looking on the above uniqueness-theorems one could expect that in the case of a full subcategory $B \subseteq A$ and a topological functor $U : A \to \text{Set}$ the only chance for a functor from $B$ to $\text{Set}$ to be topological is the restriction of $U$. In general this is not true. But it holds if $(A, U)$ is a $c$-construct, i.e. every constant map between (the underlying sets of) objects is a morphism.

References

