Free μ-lattices

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A μ-lattice L is a lattice with a specified set $\mathcal{A}_L$ of order preserving operators. This set contains all the polynomials and every unary operator induced by $\mathcal{A}_L$ has both a least prefix-point and a greatest postfix-point. A morphism of lattices is a μ-lattice morphism if it also preserves these fix-points.

The subject of this talk is the functor $\mathcal{J}$, left adjoint to the forgetful functor U from the category of μ-lattices to that of ordered sets. Given an ordered set P the μ-lattice $\mathcal{J}_P$ is described as a lattice of games with possibly infinite plays. The characterisation of fix-points by games was suggested by two constructions related to the theory of Rabin chain games. I shall discuss games in $\mathcal{J}_P$ and the way of characterising the order relation in $\mathcal{J}_P$ by saying that $G \leq H$ if there exists a winning strategy for one player, the mediator, in a compound game of communication $[G, H]$. Using techniques from game theory I'll describe a decision method, relative to P, for the order relation in $\mathcal{J}_P$.

Eventually I'll discuss the technique used to prove freeness of $\mathcal{J}_P$, which I consider to be the main achievement of this research. The analogy between strategies and proofs led to compare a bounded memory strategy to a circular proof: with respect to the usual notion of proof, i.e. a tree labelled by sequents, in circular proofs graphs replace trees. These graphs are possibly cyclic and they satisfy conditions on cycles similar to those for infinite tableaus in the theory of μ-calculus. The fact that $\mathcal{J}_P$ is a μ-lattice can be interpreted by saying that circular proofs provide a perfectly adequate cut-free presentation of a μ-lattice. The proof of freeness of $\mathcal{J}_P$ provides a method to translate circular proofs into more usual tree-like proofs, showing that the presentation by circular proofs is not a stronger system than the obvious presentation by trees.

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