Introduction to linear bicategories

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Linear bicategories are a generalization of the notion of a bicategory, in which the one horizontal composition is replaced by two (linked) horizontal compositions. These compositions provide a semantic model for the tensor and par of linear logic: in particular, as composition is fundamentally noncommutative, they provides a suggestive source of models for noncommutative linear logic.

In a linear bicategory, the logical notion of complementation becomes a natural linear notion of adjunction. Just as ordinary adjoints are related to (Kan) extensions, these linear adjoints are related to the appropriate notion of linear extension.

There is also a stronger notion of complementation, which arises, for example, in cyclic linear logic. This sort of complementation is modelled by cyclic adjoints. This leads to the notion of a ∗-linear bicategory and the coherence conditions which it must satisfy. Cyclic adjoints also give rise to linear monads: these are, essentially, the appropriate generalization (to the linear setting) of Frobenius algebras.

A number of examples of linear bicategories arising from different sources are described, and a number of constructions which result in linear bicategories are indicated.