Survey of the Abstract Stone Duality

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Stone Duality is a re-axiomatisation of general topology without arbitrary joins, so that it also axiomatises recursion theory. The idea (presented at Tours in 1993) is to take Pari's theorem, that the contravariant powerset functor in a topos is self-adjoint and monadic, as an axiom for the open-subset lattice. This didn't get very far until I discovered the Euclidean principle (presented at Vancouver in 1997) that $\phi \wedge F(\phi) = \phi \wedge F(\top)$, the dual of which also holds in topology and recursion, though only for Boolean toposes.

I shall present a statement of the completed results of the paper ("An Abstract Stone Duality, I") that I announced in January and is now being refereed, together with my plan for the development of this programme, including results for which I have proofs or just ideas.

The completed paper investigates the force of the monadic and Euclidean properties, and then gives synthetic definitions of open, discrete, compact and Hausdorff spaces in terms of internal quantifiers and in/equality. It proves that the full subcategories of open discrete and compact Hausdorff spaces form pretoposes, and concludes with the "converse" of Pari's theorem, that a category of this kind with all such quantifiers is an elementary topos.

The other work includes the construction of such categories, lifting, partial products, domain theory, the real numbers and a possible imperative interpretation using continuations.