Geometry of reference frames in relativity

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In a given spacetime $(\mathcal{V}_4, g)$ reference frame consists of:

(a) A reference space, that is, a 3-parameter congruence of timelike worldlines in $\mathcal{V}_4$ or, equivalently, the unitary tangent field, u,

(b) a time scale, defined by a 1-form $\theta \in \Lambda^1\mathcal{V}_4$ such that $\langle\theta, u\rangle \neq 0$, and

(c) a spatial $\overline{g}$, i. e., a covariant symmetric tensor such that: $\mathrm{Rad}(\overline{g}) = \mathrm{span}[u]$ and positive on $\mathrm{Ker}\,\theta$.

A reference frame induces on spacetime a geometric structure that is analogous in many respects to a Riemannian structure induced by g. Namely, an adapted linear connexion, a curvature tensor, etc. Furthermore, the case of a "sub-frame of reference" yields some results that are prety similar to Gauss and Codazzi-Mainardi equations that are obtained for Riemannian submanifolds.

This elaboration permits to fill a remarkable theoretical lack. In experiments dessigned to detect an anisotropy in the speed of light in vacuo, the experimentalists sets up a resonant cavity made of matterials as rigid as possible and tacitly assumes that the cavity's geometry does not change when it is moved around. In the theoretical level this means that the spatial metric $\overline{g}$ embodied by the cavity is rigid and has the property of free mobility —hence, it has constant curvature.

To advance a relationship between this metric $\overline{g}$, on one hand, and the spacetime metric g and the vector field u (i.e., the motion of the reference space), on the other, actually implies a hypothesis on the behaviour of the materials the cavity is made of. Nevertheless, not any relationship is viable, e.g., the infinitesimal radar metric [1] cannot be taken for $\overline{g}$. Indeed, the rigidity of radar metric imposes very strong restrictions on both the spacetime and the reference motion [2].

We finally propose a slight modification of the radar metric (the so called constrictions transformations) that does not present the above commented fatal obstructions.

References

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