The collinearity equations:

$$x = x_p - c\frac{r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)}{r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)}$$

$$y = y_p - c\frac{r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)}{r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)}$$

Let's use the following abbreviations:

$$N_x = r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)$$

$$N_y = r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)$$

$$D = r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)$$

Partial derivatives with respect to interior orientation parameters; (x_p, y_p, c):

The x-coordinate equation

$$\frac{\partial x}{\partial x_p} = 1 \qquad \frac{\partial x}{\partial y_p} = 0 \qquad \frac{\partial x}{\partial c} = -\frac{r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)}{r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)} = -\frac{N_x}{D}$$

The y-coordinate equation

$$\frac{\partial y}{\partial x_p} = 0 \qquad \frac{\partial y}{\partial y_p} = 1 \qquad \frac{\partial y}{\partial c} = -\frac{r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)}{r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)} = -\frac{N_y}{D}$$

Partial derivatives with respect to exterior orientation parameters; ground coordinates (X_o, Y_o, Z_o):

The x-coordinate equation

$$\frac{\partial x}{\partial X_o} = c\frac{r_{11}\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]-r_{13}\left[r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$\frac{\partial x}{\partial Y_o} = c\frac{r_{21}\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]-r_{23}\left[r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$\frac{\partial x}{\partial Z_o} = c\frac{r_{31}\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]-r_{33}\left[r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

The y-coordinate equation

$$\frac{\partial y}{\partial X_o} = c\frac{r_{12}\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]-r_{13}\left[r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$\frac{\partial y}{\partial Y_o} = c\frac{r_{22}\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]-r_{23}\left[r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$\frac{\partial y}{\partial Z_o} = c\frac{r_{32}\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]-r_{33}\left[r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

In abbreviated form

$$\frac{\partial x}{\partial X_o} = c\frac{r_{11}D - r_{13}N_x}{D^2} \qquad \frac{\partial x}{\partial Y_o} = c\frac{r_{21}D - r_{23}N_x}{D^2} \qquad \frac{\partial x}{\partial Z_o} = c\frac{r_{31}D - r_{33}N_x}{D^2}$$

$$\frac{\partial y}{\partial X_o} = c\frac{r_{12}D - r_{13}N_y}{D^2} \qquad \frac{\partial y}{\partial Y_o} = c\frac{r_{22}D - r_{23}N_y}{D^2} \qquad \frac{\partial y}{\partial Z_o} = c\frac{r_{32}D - r_{33}N_y}{D^2}$$

Partial derivatives with respect to exterior orientation parameters; attitude of image plane (ω, ϕ, κ):

The rotation matrix R is:

$r_{11} = \cos\phi\cos\kappa$	$r_{12} = -\cos\phi\sin\kappa$	$r_{13} = \sin\phi$
$r_{21} = \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa$	$r_{22} = \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa$	$r_{23} = -\sin\omega\cos\phi$
$r_{31} = \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa$	$r_{32} = \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa$	$r_{33} = \cos\omega\cos\phi$

$\frac{\partial(Element)}{\partial\omega}$	$\frac{\partial(Element)}{\partial\phi}$	$\frac{\partial(Element)}{\partial\kappa}$
$\frac{\partial r_{11}}{\partial\omega} = 0$	$\frac{\partial r_{11}}{\partial\phi} = -\sin\phi\cos\kappa$	$\frac{\partial r_{11}}{\partial\kappa} = -\cos\phi\sin\kappa = r_{12}$
$\frac{\partial r_{21}}{\partial\omega} = -\sin\omega\sin\kappa + \cos\omega\sin\phi\cos\kappa = -r_{31}$	$\frac{\partial r_{21}}{\partial\phi} = \sin\omega\cos\phi\cos\kappa$	$\frac{\partial r_{21}}{\partial\kappa} = \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa = r_{22}$
$\frac{\partial r_{31}}{\partial\omega} = \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa = r_{21}$	$\frac{\partial r_{31}}{\partial\phi} = -\cos\omega\cos\phi\cos\kappa$	$\frac{\partial r_{31}}{\partial\kappa} = \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa = r_{32}$
$\frac{\partial r_{12}}{\partial\omega} = 0$	$\frac{\partial r_{12}}{\partial\phi} = \sin\phi\sin\kappa$	$\frac{\partial r_{12}}{\partial\kappa} = -\cos\phi\cos\kappa = -r_{11}$
$\frac{\partial r_{22}}{\partial\omega} = -\sin\omega\cos\kappa - \cos\omega\sin\phi\sin\kappa = -r_{32}$	$\frac{\partial r_{22}}{\partial\phi} = -\sin\omega\cos\phi\sin\kappa$	$\frac{\partial r_{22}}{\partial\kappa} = -\cos\omega\sin\kappa - \sin\omega\sin\phi\cos\kappa = -r_{21}$
$\frac{\partial r_{32}}{\partial\omega} = \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa = r_{22}$	$\frac{\partial r_{32}}{\partial\phi} = \cos\omega\cos\phi\sin\kappa$	$\frac{\partial r_{32}}{\partial\kappa} = -\sin\omega\sin\kappa + \cos\omega\sin\phi\cos\kappa = -r_{31}$
$\frac{\partial r_{13}}{\partial\omega} = 0$	$\frac{\partial r_{13}}{\partial\phi} = \cos\phi$	$\frac{\partial r_{13}}{\partial\kappa} = 0$
$\frac{\partial r_{23}}{\partial\omega} = -\cos\omega\cos\phi = -r_{33}$	$\frac{\partial r_{23}}{\partial\phi} = \sin\omega\sin\phi$	$\frac{\partial r_{23}}{\partial\kappa} = 0$
$\frac{\partial r_{33}}{\partial\omega} = -\sin\omega\cos\phi = r_{23}$	$\frac{\partial r_{33}}{\partial\phi} = -\cos\omega\sin\phi$	$\frac{\partial r_{33}}{\partial\kappa} = 0$

The x-coordinate equation

$$\frac{\partial x}{\partial \omega} = -c\frac{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]\left[0-r_{31}\cdot(Y-Y_o)+r_{21}\cdot(Z-Z_o)\right]+\left[r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)\right]\left[0+r_{33}\cdot(Y-Y_o)-r_{23}\cdot(Z-\right.}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$\frac{\partial x}{\partial \omega} = -c\frac{D\left[-r_{31}\cdot(Y-Y_o)+r_{21}\cdot(Z-Z_o)\right]+N_x\left[r_{33}\cdot(Y-Y_o)-r_{23}\cdot(Z-Z_o)\right]}{D^2}$$

$$\frac{\partial x}{\partial \phi} = -c\frac{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]\left[-\sin\phi\cos\kappa\cdot(X-X_o)+\sin\omega\cos\phi\cos\kappa\cdot(Y-Y_o)-\cos\omega\cos\phi\cos\kappa\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$-c\frac{\left[r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)\right]\left[\cos\phi\cdot(X-X_o)+\sin\omega\sin\phi\cdot(Y-Y_o)-\cos\omega\sin\phi\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$\frac{\partial x}{\partial \phi} = -c\frac{D\left[-\sin\phi\cos\kappa\cdot(X-X_o)+\sin\omega\cos\phi\cos\kappa\cdot(Y-Y_o)-\cos\omega\cos\phi\cos\kappa\cdot(Z-Z_o)\right]+N_x\left[\cos\phi\cdot(X-X_o)+\sin\omega\sin\phi\cdot(Y-Y_o)-\cos\omega\sin\phi\cdot(Z-Z_o)\right]}{D^2}$$

$$\frac{\partial x}{\partial \phi} = -c\frac{-D\cos\kappa\left[-\sin\phi\cdot(X-X_o)+\sin\omega\cos\phi\cdot(Y-Y_o)-\cos\omega\cos\phi\cdot(Z-Z_o)\right]+N_x\left[\cos\phi\cdot(X-X_o)+\sin\omega\sin\phi\cdot(Y-Y_o)-\cos\omega\sin\phi\cdot(Z-Z_o)\right]}{D^2}$$

$$\frac{\partial x}{\partial \phi} = -c\frac{-D^2\cos\kappa+N_x\left[\cos\phi\cdot(X-X_o)+\sin\omega\sin\phi\cdot(Y-Y_o)-\cos\omega\sin\phi\cdot(Z-Z_o)\right]}{D^2}$$

$$\frac{\partial x}{\partial \phi} = -c\frac{-D^2\cos\kappa+N_x\left[-N_x\cdot\cos\kappa+N_y\sin\kappa\right]}{D^2}$$

$$\frac{\partial x}{\partial \kappa} = -c\frac{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]\left[r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$\frac{\partial x}{\partial \kappa} = -c\frac{N_y}{D}$$

In Summary:

$$\frac{\partial x}{\partial \omega} = -c\frac{D\left[-r_{31}\cdot(Y-Y_o)+r_{21}\cdot(Z-Z_o)\right]+N_x\left[r_{33}\cdot(Y-Y_o)-r_{23}\cdot(Z-Z_o)\right]}{D^2} \qquad \frac{\partial x}{\partial \phi} = -c\frac{-D^2\cos\kappa+N_x\left[-N_x\cdot\cos\kappa+N_y\sin\kappa\right]}{D^2} \qquad \frac{\partial x}{\partial \kappa} = -c\frac{N_y}{D}$$

The y-coordinate equation

$$\frac{\partial y}{\partial \omega} = -c\frac{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]\left[0-r_{32}\cdot(Y-Y_o)+r_{22}\cdot(Z-Z_o)\right]+\left[r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)\right]\left[0+r_{33}\cdot(Y-Y_o)-r_{23}\right.}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$\frac{\partial y}{\partial \omega} = -c\frac{D\left[-r_{32}\cdot(Y-Y_o)+r_{22}\cdot(Z-Z_o)\right]+N_y\left[r_{33}\cdot(Y-Y_o)-r_{23}\cdot(Z-Z_o)\right]}{D^2}$$

$$\frac{\partial y}{\partial \phi} = -c\frac{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]\left[\sin\phi\sin\kappa\cdot(X-X_o)-\sin\omega\cos\phi\sin\kappa\cdot(Y-Y_o)+\cos\omega\cos\phi\sin\kappa\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$-c\frac{\left[r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)\right]\left[\cos\phi\cdot(X-X_o)+\sin\omega\sin\phi\cdot(Y-Y_o)-\cos\omega\sin\phi\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$\frac{\partial y}{\partial \phi} = -c\frac{D\left[\sin\phi\sin\kappa\cdot(X-X_o)-\sin\omega\cos\phi\sin\kappa\cdot(Y-Y_o)+\cos\omega\cos\phi\sin\kappa\cdot(Z-Z_o)\right]+N_y\left[\cos\phi\cdot(X-X_o)+\sin\omega\sin\phi\cdot(Y-Y_o)-\cos\omega\sin\phi\cdot(Z-Z_o)\right]}{D^2}$$

$$\frac{\partial y}{\partial \phi} = -c\frac{D\sin\kappa\left[\sin\phi\cdot(X-X_o)-\sin\omega\cos\phi\cdot(Y-Y_o)+\cos\omega\cos\phi\cdot(Z-Z_o)\right]+N_y\left[\cos\phi\cdot(X-X_o)+\sin\omega\sin\phi\cdot(Y-Y_o)-\cos\omega\sin\phi\cdot(Z-Z_o)\right]}{D^2}$$

$$\frac{\partial y}{\partial \phi} = -c\frac{D^2\sin\kappa+N_y\left[\cos\phi\cdot(X-X_o)+\sin\omega\sin\phi\cdot(Y-Y_o)-\cos\omega\sin\phi\cdot(Z-Z_o)\right]}{D^2}$$

$$\frac{\partial y}{\partial \phi} = -c\frac{D^2\sin\kappa+N_y\left[N_x\cos\kappa+N_y\sin\kappa\right]}{D^2}$$

$$\frac{\partial y}{\partial \kappa} = -c\frac{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]\left[-r_{11}\cdot(X-X_o)-r_{21}\cdot(Y-Y_o)-r_{31}\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$\frac{\partial y}{\partial \kappa} = c\frac{N_x}{D}$$

In Summary:

$$\frac{\partial y}{\partial \omega} = -c\frac{D\left[-r_{32}\cdot(Y-Y_o)+r_{22}\cdot(Z-Z_o)\right]+N_y\left[r_{33}\cdot(Y-Y_o)-r_{23}\cdot(Z-Z_o)\right]}{D^2} \quad \frac{\partial y}{\partial \phi} = -c\frac{D^2\sin\kappa+N_y\left[N_x\cos\kappa+N_y\sin\kappa\right]}{D^2} \quad \frac{\partial y}{\partial \kappa} = c\frac{N_x}{D}$$

Partial derivatives with respect to ground coordinates of tie points(X, Y, Z)

The x-coordinate equation

$$\frac{\partial x}{\partial X} = -\frac{\partial x}{\partial X_o} = -c\frac{r_{11}\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]-r_{13}\left[r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$\frac{\partial x}{\partial Y} = -\frac{\partial x}{\partial Y_o} = -c\frac{r_{21}\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]-r_{23}\left[r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$\frac{\partial x}{\partial Z} = -\frac{\partial x}{\partial Z_o} = -c\frac{r_{31}\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]-r_{33}\left[r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

The y-coordinate equation

$$\frac{\partial y}{\partial X} = -\frac{\partial y}{\partial X_o} = -c\frac{r_{12}\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]-r_{13}\left[r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$\frac{\partial y}{\partial Y} = -\frac{\partial y}{\partial Y_o} = -c\frac{r_{22}\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]-r_{23}\left[r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

$$\frac{\partial y}{\partial Z} = -\frac{\partial y}{\partial Z_o} = -c\frac{r_{32}\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]-r_{33}\left[r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)\right]}{\left[r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)\right]^2}$$

In abbreviated form

$$\frac{\partial x}{\partial X} = -c\frac{r_{11}D-r_{13}N_x}{D^2} \qquad \frac{\partial x}{\partial Y} = -c\frac{r_{21}D-r_{23}N_x}{D^2} \qquad \frac{\partial x}{\partial Z} = -c\frac{r_{31}D-r_{33}N_x}{D^2}$$

$$\frac{\partial y}{\partial X} = -c\frac{r_{12}D-r_{13}N_y}{D^2} \qquad \frac{\partial y}{\partial Y} = -c\frac{r_{22}D-r_{23}N_y}{D^2} \qquad \frac{\partial y}{\partial Z} = -c\frac{r_{32}D-r_{33}N_y}{D^2}$$