ENGO 431 – Analytical Photogrammetry

Fall 2004

LAB 4: SINGLE PHOTO RESECTION

Due date: November 22nd 2004

Objective:

Determine the Exterior Orientation Parameters (EOP) of a single photo using least squares adjustment procedure.

Given:

1. Interior Orientation Parameters (IOP) of the camera from the Camera Calibration Certificate (CCC);

   (a) Calibrated focal length (c): 153.167 mm.

   (b) Principal point coordinates in the fiducial system are:

   x_p= 0.001 mm

   y_p= -0.053 mm

2. Ground coordinates of control points:

Point ID	X (m)	Y (m)	Z (m)
110	676349.506	7184079.099	885.758
3110	676187.214	7184138.610	883.488
3111	676423.413	7184149.085	887.494
3112	676782.089	7184150.526	888.756
3113	676259.424	7184749.781	898.677
3114	676377.925	7184720.718	904.759
3115	676763.528	7184713.331	923.008
4110	677123.559	7184100.303	890.640
4111	677267.560	7184089.381	894.912
4112	677379.258	7184208.899	894.467
4113	677020.416	7184626.978	921.563
4114	677155.718	7184626.399	925.612
4115	677213.374	7184754.621	925.388
4116	677332.347	7184765.561	928.863

3. Image coordinates of control points as measured and reduced (i.e., after the removal of radial, de-centering, and atmospheric refraction errors) in Lab 3 for images 3314_50.jpg and 3316_50.jpg.

4. Use the flying height as determined in previous labs.

Single Photo Resection (SPR)

The objective of single photo resection is to determine the position of the perspective center and the orientation of the image coordinate system (EOPs) relative to the ground coordinate system using only the image under consideration.

The solution to the single photo resection problem is based on the Collinearity equations and is performed for each photo independently.

$$x = x_p - c\frac{r_{11}\cdot(X - X_o) + r_{21}\cdot(Y - Y_o) + r_{31}\cdot(Z - Z_o)}{r_{13}\cdot(X - X_o) + r_{23}\cdot(Y - Y_o) + r_{33}\cdot(Z - Z_o)} = x_p - c\,\frac{N_x}{D}$$

$$y = y_p - c\frac{r_{12}\cdot(X - X_o) + r_{22}\cdot(Y - Y_o) + r_{32}\cdot(Z - Z_o)}{r_{13}\cdot(X - X_o) + r_{23}\cdot(Y - Y_o) + r_{33}\cdot(Z - Z_o)} = y_p - c\,\frac{N_y}{D}$$

with:

$$N_x = r_{11}\cdot(X - X_o) + r_{21}\cdot(Y - Y_o) + r_{31}\cdot(Z - Z_o)$$

$$N_y = r_{12}\cdot(X - X_o) + r_{22}\cdot(Y - Y_o) + r_{32}\cdot(Z - Z_o)$$

$$D = r_{13}\cdot(X - X_o) + r_{23}\cdot(Y - Y_o) + r_{33}\cdot(Z - Z_o)$$

Collinearity equations are nonlinear with respect to the Exterior Orientation Parameters (EOP), which are the unknown parameters of the single photo resection problem:

- The three rotation angles (ω, ϕ, κ), which are inherent in the elements of the rotation matrix. These angles should be applied to the ground coordinate system until it is parallel to the image coordinate system.
- The position of the exposure station (X_o, Y_o, Z_o) relative to ground coordinate system.

Assumptions

For this lab, the following quantities are considered as known quantities and treated as constants:

- The Interior Orientation Parameters (IOP), x_p, y_p, c.
- The ground coordinates of the control points (X, Y, Z).

To solve for the six unknown Exterior Orientation Parameters (EOP), a minimum of three ground control points is required (two collinearity equations per point). It is recommended to use more or all of the available control points in order to exploit the added accuracy provided by data redundancy. This is achieved through a least squares procedure.

Preamble

For the sake of implementing a least squares adjustment solution, the collinearity equations have to be linearized with respect to the unknown parameters (Exterior Orientation Parameters - EOP) using Taylor's theorem. Complete partial derivatives of the collinearity equations are posted on the course website. Though, only those pertaining to the assumed unknowns will be used.

The linearized form of the equations becomes:

$$x = x_o + \left(\frac{\partial x}{\partial X_o}\right)_o dX_o + \left(\frac{\partial x}{\partial Y_o}\right)_o dY_o + \left(\frac{\partial x}{\partial Z_o}\right)_o dZ_o + \left(\frac{\partial x}{\partial \omega}\right)_o d\omega + \left(\frac{\partial x}{\partial \phi}\right)_o d\phi + \left(\frac{\partial x}{\partial \kappa}\right)_o d\kappa$$

$$y = y_o + \left(\frac{\partial y}{\partial X_o}\right)_o dX_o + \left(\frac{\partial y}{\partial Y_o}\right)_o dY_o + \left(\frac{\partial y}{\partial Z_o}\right)_o dZ_o + \left(\frac{\partial y}{\partial \omega}\right)_o d\omega + \left(\frac{\partial y}{\partial \phi}\right)_o d\phi + \left(\frac{\partial y}{\partial \kappa}\right)_o d\kappa$$

where x_o and y_o are the evaluated x and y image coordinates using the initial approximations of the unknowns parameters.

$\left(\frac{\partial x}{\partial X_o}\right)_o, \left(\frac{\partial x}{\partial Y_o}\right)_o$, ... etc., are the partial derivatives of x and y with respect to the indicated unknowns evaluated at the initial approximations of these parameters.

dX_o, dY_o, ... etc., are the unknown corrections to be applied to the initial approximations.

It can be noted that only the first order terms are used. The truncation of higher order terms will be compensated for by the iterative procedure.

In the above linearized collinearity equations, the partial derivatives can be replaced by a simpler notation for handling convenience, as follows:

$$a_1 = \left(\frac{\partial x}{\partial X_o}\right)_o, a_2 = \left(\frac{\partial x}{\partial Y_o}\right)_o, a_3 = \left(\frac{\partial x}{\partial Z_o}\right)_o, a_4 = \left(\frac{\partial x}{\partial \omega}\right)_o, a_5 = \left(\frac{\partial x}{\partial \phi}\right)_o, a_6 = \left(\frac{\partial x}{\partial \kappa}\right)_o$$

$$b_1 = \left(\frac{\partial y}{\partial X_o}\right)_o, b_2 = \left(\frac{\partial y}{\partial Y_o}\right)_o, b_3 = \left(\frac{\partial y}{\partial Z_o}\right)_o, b_4 = \left(\frac{\partial y}{\partial \omega}\right)_o, b_5 = \left(\frac{\partial y}{\partial \phi}\right)_o, b_6 = \left(\frac{\partial y}{\partial \kappa}\right)_o$$

which yields:

$$x - x_o = a_1\ dX_o + a_2\ dY_o + a_3\ dZ_o + a_4\ d\omega + a_5\ d\phi + a_6\ d\kappa$$

$$y - y_o = b_1\ dX_o + b_2\ dY_o + b_3\ dZ_o + b_4\ d\omega + b_5\ d\phi + b_6\ d\kappa$$

This final form linearized collinearity equations will be used to illustrate the various steps of the solution.

Least Squares Procedure:

The solution now proceeds by building the Gauss-Markov (Observation Equations) Model which will be used to solve for the unknown EOPs.

The general form of the observation equations model is

$$y = A\,x + e \qquad e \sim (0, \sigma_o^2\, P^{-1})$$

The terms in the above equations have different definitions based on whether the system is linear in nature or have been linearized,

Linear System	Linearized System
y: is the observations vector	y: is the vector of differences between the measured and computed image coordinates using the approximate values for the unknown parameters
A: is the design matrix	A: is the design matrix composed of the partial derivatives
x: is the vector of unknown parameters	x: is the vector of unknown corrections to the approximate values of the Exterior Orientation Parameters
e: is the error vector	e: is the error vector

In our case, of course, the terms are defined as in the second column.

Balance between the observations and unknown parameters

Number of observations, n

Number of points × 2 coordinates/point

Number of parameters, m = 6

$dX_o\ dY_o\ dZ_o\ d\omega\ d\phi\ d\kappa$

Redundancy, r = n – m;

The observation equations can be expressed in a matrix form as follows:

$$y = \begin{bmatrix} x_1 - x_{1_o} \\ y_1 - y_{1_o} \\ x_2 - x_{2_o} \\ y_2 - y_{2_o} \\ \vdots \\ \vdots \\ x_{n/2} - x_{n/2_o} \\ y_{n/2} - y_{n/2_o} \end{bmatrix}_{n\times 1} = \begin{bmatrix} a_{1_1} & a_{2_1} & a_{3_1} & a_{4_1} & a_{5_1} & a_{6_1} \\ b_{1_1} & b_{2_1} & b_{3_1} & b_{4_1} & b_{5_1} & b_{6_1} \\ a_{1_2} & a_{2_2} & a_{3_2} & a_{4_2} & a_{5_2} & a_{6_2} \\ b_{1_2} & b_{2_2} & b_{3_2} & b_{4_2} & b_{5_2} & b_{6_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}_{n\times 6} \cdot \begin{bmatrix} dX_o \\ dY_o \\ dZ_o \\ d\omega \\ d\phi \\ d\kappa \end{bmatrix}_{6\times 1} + \begin{bmatrix} e_{x_1} \\ e_{y_1} \\ e_{x_2} \\ e_{y_2} \\ \vdots \\ \vdots \\ e_{x_{n/2}} \\ e_{y_{n/2}} \end{bmatrix}_{n\times 1}$$

Now, let's determine the involved values in the above equations:

<u>The differences vector, y:</u>

$$x_{measured} - x_o = x_{measured} - \left(x_p - c\frac{N_x}{D} \right)_o$$

$$y_{measured} - y_o = y_{measured} - \left(y_p - c\frac{N_y}{D} \right)_o$$

where $x_{measured}$ and $y_{measured}$ are imported from the outcome of Lab-3.

To calculate the above values, a set of initial approximations for the unknown parameters should be obtained.

<u>Initial Approximations of EOPs:</u>

a. The attitude angles (ω, ϕ, κ).

- A vertical photograph is assumed (i.e., $\omega = \phi = 0$).
- The κ angle will be estimated from the following procedure:
  - Estimate the parameters of a 2-D similarity transformation model from a least squares adjustment procedure using the measured (x, y) image coordinates and the corresponding (X, Y) ground coordinates.

    $$X = a_0 + a\,x - b\,y$$

    $$Y = b_0 + b\,x + a\,y$$

  - Calculate $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ and use it as an initial approximation of κ. Use full circle tangent (atan2 in Matlab)

Now the elements of the R matrix can be calculated.

$r_{11} = \cos\phi\cos\kappa$	$r_{12} = -\cos\phi\sin\kappa$	$r_{13} = \sin\phi$
$r_{21} = \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa$	$r_{22} = \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa$	$r_{23} = -\sin\omega\cos\phi$
$r_{31} = \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa$	$r_{32} = \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa$	$r_{33} = \cos\omega\cos\phi$

b. X_o, Y_o, Z_o.

- The initial approximations of X_o, Y_o are taken as the a_o and b_o values of the above 2-D similarity model.
- For Z_o use H (or H + Z_{avg} of ground control points) as the first approximation for Z_o

c. The X, Y, Z values are the given ground coordinates of control points.

<u>The design matrix, A:</u>

- The rows of the design matrix are calculated from the partial derivative terms of the collinearity equations with respect to the Exterior Orientation Parameters.
- The following terms $a_1, \ldots, a_6$ and $b_1, \ldots, b_6$ will constitute the elements of the two rows of the A matrix associated with each point. The same approximate values for the rotation angles and coordinates are used for computing the A matrix.

$$a_1 = \frac{\partial x}{\partial X_o} = c\,\frac{r_{11}D - r_{13}N_x}{D^2} \qquad a_2 = \frac{\partial x}{\partial Y_o} = c\,\frac{r_{21}D - r_{23}N_x}{D^2} \qquad a_3 = \frac{\partial x}{\partial Z_o} = c\,\frac{r_{31}D - r_{33}N_x}{D^2}$$

$$a_4 = \frac{\partial x}{\partial \omega} = -c\,\frac{D\left[-r_{31}\cdot(Y - Y_o) + r_{21}\cdot(Z - Z_o)\right] + N_x\left[r_{33}\cdot(Y - Y_o) - r_{23}\cdot(Z - Z_o)\right]}{D^2}$$

$$a_5 = \frac{\partial x}{\partial \phi} = -c\,\frac{-D^2\cos\kappa + N_x\left[-N_x\cdot\cos\kappa + N_y\sin\kappa\right]}{D^2} \qquad a_6 = \frac{\partial x}{\partial \kappa} = -c\,\frac{N_y}{D}$$

$$b_1 = \frac{\partial y}{\partial X_o} = c\frac{r_{12}D - r_{13}N_y}{D^2} \qquad b_2 = \frac{\partial y}{\partial Y_o} = c\frac{r_{22}D - r_{23}N_y}{D^2} \qquad b_3 = \frac{\partial y}{\partial Z_o} = c\frac{r_{32}D - r_{33}N_y}{D^2}$$

$$b_4 = \frac{\partial y}{\partial \omega} = -c\frac{D\left[-r_{32}\cdot(Y - Y_o) + r_{22}\cdot(Z - Z_o)\right] + N_y\left[r_{33}\cdot(Y - Y_o) - r_{23}\cdot(Z - Z_o)\right]}{D^2}$$

$$b_5 = \frac{\partial y}{\partial \phi} = -c\frac{D^2\sin\kappa + N_y\left[N_x\cos\kappa + N_y\sin\kappa\right]}{D^2} \qquad b_6 = \frac{\partial y}{\partial \kappa} = c\frac{N_x}{D}$$

Required Task

Develop a computer program using Matlab, C++, or any language which you prefer to solve for the exterior orientation parameters. Implement your program to estimate the EOP for images 3314_50.jpg and 3316_50.jpg.

Deliverables and Report Preparation

Your lab report should include the following for both images (3314_50.jpg & 3316-50.jpg):

- Measured image coordinates and the approximations to the unknowns indicating any assumptions made.
- The modified Exterior Orientation Parameters after each iteration.
- The final adjusted values of the Exterior Orientation Parameters.
- An estimate of the variance component.
- The posterior variance-covariance (dispersion) matrix of the parameters.
- The residuals associated with the image coordinate measurements.
- Explanation of your results and any problems encountered.