10^{th} Annual Workshop on Applications and Generalizations of Complex Analysis

CIM-Centro internacional de Matemática

Coimbra-23/06/07

Abstracts

Supported by:
CIM-Centro Internacional de Matemática
CMUC-Centro de Matemática da Universidade de Coimbra
UIMA- Unidade de Investigação Matemática e Aplicações da Universidade de Aveiro

9:45-10:15 Richard Delanghe, University of Ghent
Subclasses of monogenic functions

Let $\Omega \subset \mathbb{R}^{m+1}$ be open and let $\mathbb{R}_{0,m+1}$ be the real Clifford algebra generated by the orthonormal basis $e = (e_0, e_1, ..., e_m)$ of the quadratic vector space $\mathbb{R}^{0,m+1}$. Furthermore, let $F : \Omega \to \mathbb{R}_{0,m+1}$ be a smooth function in Ω. Then F is said to be left (resp. right) monogenic in Ω iff $\partial_x F = 0$ (resp. $F\partial_x = 0$) in Ω. Hereby $\partial_x = \sum_{i=0}^{m} e_i \partial_{x_i}$ is the Dirac operator in $\mathbb{R}^{m+1}$.
Consider the decomposition of $\mathbb{R}_{0,m+1}$ into its r-vector subspaces $\mathbb{R}_{0,m+1}^{(r)}$, i.e.

$$\mathbb{R}_{0,m+1} = \sum_{r=0}^{m+1} \oplus \, \mathbb{R}_{0,m+1}^{(r)}, \tag{1}$$

where $\mathbb{R}_{0,m+1}^{(r)} = span_{\mathbb{R}}(e_A : |A| = r)$. Hereby $e_A = e_{i_1} e_{i_2} ... e_{i_r}$, $A = \{i_1, ..., i_r\} \subset \{0, 1, ..., m\}$, is an arbitrary element of the standard basis of $\mathbb{R}_{0,m+1}$ generated by $e = (e_0, e_1, ..., e_m)$.

Moreover, let for $r \in \{0, 1, ..., m+1\}$ fixed, $\mathcal{E}_r(\Omega)$ and $\wedge^r(\Omega)$, denote, respectively, the spaces of smooth $\mathbb{R}_{0,m+1}^{(r)}$ -valued functions and of smooth r-forms in Ω. If $F_r = \sum_{|A|=r} F_{A,r}\, e_A \in \mathcal{E}_r(\Omega)$, put $\Theta\, F_r = \omega^r$ where $\omega^r = \sum_{|A|=r} \omega_{A,r} dx^A \in \wedge^r(\Omega)$ with $\omega_{A,r} = F_{A,r}$ for all A and $dx^A = dx^{i_1} \wedge dx^{i_2} \wedge ... \wedge dx^{i_r}$.

Putting $\mathcal{E}(\Omega) = \sum_{r=0}^{m+1} \oplus \mathcal{E}_r(\Omega)$ and $\wedge(\Omega) = \sum_{r=0}^{m+1} \oplus \wedge^r(\Omega)$, then extending Θ by linearity to $\mathcal{E}(\Omega)$, we have that, if $F \in \mathcal{E}(\Omega)$ and $\omega = \Theta F$,

$$\partial_x F = 0 \Leftrightarrow (d + d^*)\, \omega = 0 \tag{2}$$

Hereby d and d^* are the exterior derivative and the Hodge $*$-derivative acting on $\wedge(\Omega)$.

The following subclasses of monogemic functions will be discussed.

(i) Consider the class of $\mathbb{R}_{0,m+1}^{(r)}$-valued functions F_r in Ω, $r \in \{1, ..., m\}$ being fixed. Putting $\omega^r = \Theta F_r$, we have

$$\partial_x F_r = 0 \Longleftrightarrow \begin{cases} d\omega^r &= 0 \\ d^*\omega^r &= 0 \end{cases} \tag{3}$$

As is well known, the latter system in (3) is the Hodge-de Rham system for r-forms and its solutions are called harmonic r-forms.

(ii) Consider the class of $\mathbb{R}_{0,m+1}^{(r)} \oplus \mathbb{R}_{0,m+1}^{(r+2)}$-valued functions $W = W^r + W^{r+2}$, where $W^j \in \mathcal{E}_j(\Omega), j = r, r+2, r \in \{1, ..., m\}$. Putting $\omega = \omega^r + \omega^{r+2} = \Theta W$, we have

$$\begin{aligned} \partial_x W = 0 \quad &\Longleftrightarrow \quad (d + d^*)\omega = 0 \\ &\Longleftrightarrow \quad \begin{cases} d^*\omega^r &= 0 \\ d\omega^r + d^*\omega^{r+2} &= 0 \\ d\omega^{r+2} &= 0 \end{cases} \end{aligned} \tag{4}$$

The latter system in (4) is known as the Moisil-Théodoresco system in $\Omega \subset \mathbb{R}^{m+1}$.

Special attention is payed to the Moisil-Théodoresco system in $\mathbb{R}^3$.

Structure theorems are proved for the elements belonging to each of the subclasses of monogenic functions. The results presented are joint work with F. Sommen (i) and J. Bory-Reyes (ii) .

10:15-10:45 Bram De Knock, University of Ghent
A Hermitean Cauchy integral formula in Clifford analysis

Orthogonal Clifford analysis is a higher dimensional function theory offering a refinement of classical harmonic analysis. The theory is centered around the concept of monogenic functions, i.e., null solutions of a first order vector valued rotation invariant differential operator, called the Dirac operator. More recently, Hermitean Clifford analysis has emerged as a new and successful branch of Clifford analysis, offering yet a refinement of the orthogonal case; it focusses on the simultaneous null solutions, called Hermitean monogenic functions, of two Hermitean Dirac operators which are invariant under the action of the unitary group. In orthogonal Clifford analysis, the Clifford–Cauchy integral formula has proven to be a corner stone of the function theory, as is the case for the traditional Cauchy formula for holomorphic functions in the complex plane. In this talk, a Hermitean Clifford -Cauchy integral formula is established by means of a matrix approach. The traditional Martinelli -Bochner formula for holomorphic functions of several complex variables is recovered as a special case.

Coffee break

11:00-11:30 Eduardo Godoy, University of Vigo
Multivariable generalized Bernstein polynomials

We introduce multivariable generalized Bernstein polynomials which generalize the classical multivariate Bernstein and discrete Bernstein polynomials. Basic properties of the new polynomials are given, including recurrence relations, q-differentiation rules and de Casteljau algorithm.Connection formulae between particular forms of these polynomials and bivariate orthogonal q-Jacobi, or q-Hahn polynomials, are presented.

11:30-12:00 Luis Garza, University Carlos III of Madrid
Spectral transformations of measures supported on the unit circle and the Szegö transformation

In this paper we analyze spectral transformations of measures supported on the unit circle with real moments. The connection with spectral transformations of measures supported on the interval $[-1, 1]$ using the Szegő transformation is presented. Some numerical examples are studied.
Keywords Measures on the unit circle, orthogonal polynomials, Carathéodory functions, spectral transformations, LU factorization.

12:00-12:30 Herbert Dueñas, University Carlos III of Madrid
The complex Toda lattice: The Bäcklund transformation and the structure of the space of solutions

In this contribution we study the second order linear differential equation satisfied by polynomials orthogonal with respect to the linear functional

$$\langle \widetilde{\mu}, p \rangle = \int_{-1}^{1} p(x)\,(1-x)^{\alpha}\,(1+x)^{\beta}\,dx + Mp(1)$$

where $\alpha > -1, \beta > -1$, $M \in \mathbb{R}_+$, and p is a polynomial with real coefficients. We also find some results concerning the distribution of their zeros. Finally, an electrostatic interpretation of the zeros in terms of a logarithmic potential with an external field is presented.

Keywords and Phrases: Orthogonal polynomials, Jabobi weights, holonomic equation, zeros, logarithmic potential.

Lunch

14:00-14:30 Rogério Serôdio, University of Beira Interior
Quasideterminants and some applications

It is well known that determinant theory plays an important role in Matrix theory with elements in a field. But if we extend the matrix theory over a skewfield the notion of determinant does not exist. However the notion os quasideterminant generalizes the determinant concept to the skewfields. Our intent here is to present the concept of quasideterminants and to apply it to the investigation of the singularity of Vandermonde matrices over a skewfield.

14:30-15:00 Nelson Vieira, University of Aveiro
Numerical Analysis for the in-stationary Schrödinger equation by means of Clifford Analysis

The in-stationary Schrödinger equation is one the basic equations of quantum theory. It plays an exceptionally important role in modern physics in that sense that it provides the basic model for phenomena as propagation of a laser beam in a medium, water waves at the free surface and plasma waves. In this talk we will construct a discrete fundamental solution for the in-stationary Schrödinger operator based on Clifford analysis techniques. With such fundamental solution we will construct a discrete counterpart for the classic operator calculus, namely we establish discrete versions for the Teodorescu and Cauchy-Bitsadze operators and the Bergman projectors. We finalize this presentation with convergence results regarding the proposed numerical method and some concrete numerical examples.

15:00-15:30 Nelson Faustino, University of Aveiro
Discrete Dirac operators on combinatorial surfaces

Discrete function theory is essentially based on discretizations of Cauchy-Riemann/ Dirac operators which preserve certain structural properties such as conformal invariance. Hereby, the combinatorics of Z^n is given up in favor of arbitrary graphs with rhombic faces (i.e. diamonds) and cell decompositions of a manifold, such as cubical and simplicial ones, are associated with certain open covers which can be assumed to be finite.

In this talk we will present a combinatorial approach to derive discrete Dirac operators on graphs/polytopes.

Starting with the space of chains (the combinatorial surface elements), we construct the combinatorial analogues of differential forms using Whitney's approach. These k-forms are essentially simplicial cochains.

Since the combinatorial Hodge star operator allows us to split $k-$forms in i and $-i$ eigenspaces, the resulting projection of $1-$forms allow us to derive combinatorial analogues of Dirac/Cauchy-Riemann operators.

The resulting Dirac operators will be compared with the discrete Dirac operators introduced recently by Faustino, Kähler and Sommen and the resulting notion of discrete monogenic functions will be compared with the notion of discrete holomorphic functions on quad-graphs.

15:30-16:00 Márcio Nascimento, Polytechnical Institute of Viseu
On orthogonal polynomials via polynomial mappings

Let $(p_n)_n$ be a given monic orthogonal polynomial sequence (OPS) and k a fixed positive integer number such that $k \geq 2$. We analyze conditions under which this OPS originates from a polynomial mapping in the following sense: to find another monic OPS $(q_n)_n$ and two polynomials π_k and θ_m, with degrees k and m (resp.), with $0 \leq m \leq k-1$, such that

$$p_{nk+m}(x) = \theta_m(x) q_n(\pi_k(x)) \quad (n = 0, 1, 2, \cdots) .$$

We determine explicitly the orthogonality measure for the given OPS $(p_n)_n$ in terms of the orthogonality measure for the OPS $(q_n)_n$. As examples, we recover several known results in the literature, including some connections between this kind of transformation laws and the spectral theory of self-adjoint Jacobi operators.

Coffee break

16:30-17:00 Antonio Durán, University of Sevilla
Orthogonal matrix polynomials satisfying second order differential equations

The theory of matrix valued orthogonal polynomials was started by M. G. Krein in 1949. They can be characterized as solutions of the difference equation

$$tP_n(t) = A_{n+1}P_{n+1}(t) + B_nP_n(t) + A_n^*P_{n-1}(t),$$

where A_n and B_n are, respectively, nonsingular and Hermitian matrices. But more than 50 years have been necessary to see the first examples of orthogonal matrix polynomials $(P_n)_n$ satisfying second order differential equations of the form

$$P_n''(t)F_2(t) + P_n'(t)F_1(t) + P_n(t)F_0 = \Gamma_n P_n(t).$$

Here F_2, F_1 and F_0 are matrix polynomials (which do not depend on n) of degrees less than or equal to 2, 1 and 0, respectively. These families of orthogonal matrix polynomials are among those that are likely to play in the case of matrix orthogonality the role of the classical families of Hermite, Laguerre and Jacobi in the case of scalar orthogonality.

The purpose of this talk is to show an overview of these examples, in particular we will discuss some of the many differences among the matrix and the scalar case, such as the (non) uniqueness of the second order differential operator.

17:00-17:30 Luís Cotrim, Polytechnical Institute of Leiria
Favard theorem in the theory of multiple orthogonal polynomials

In this talk we present the general theory of multiple orthogonal polynomials and give an interpretation of $s\,(d+1)+1$-term recurrence relation of type

$$x^s B_n = B_{n+s} + \sum_{k=0}^{s(d+1)-1} a_{n+s-1-k}^{n+s-1} B_{n+s-1-k}$$

in terms of the theory of multiple orthogonal polynomials. This enables us to give a Favard type theorem.

17:30-18:00 Maria das Neves Rebocho, University of Beira Interior
Characterizations of the Laguerre-Hahn class on the unit circle

A study about the Laguerre-Hahn class on the unit circle is presented.

Given a hermitian linear functional u and the corresponding Carathéodory function F, the functional u (or F) is said to be Laguerre-Hahn if F satisfies a Ricatti differential equation with polynomial coefficients

$$zAF' = BF^2 + CF + D \tag{5}$$

The corresponding sequence of orthogonal polynomials is called Laguerre-Hahn. The Laguerre-Hahn class of orthogonal polynomials on the unit circle includes the semi-classical orthogonal polynomials on the unit circle ($B = 0$ and C, D specific polynomials in (5)) and the Laguerre-Hahn affine orthogonal polynomials on the unit circle ($B = 0$ in (5)).

Let u be a Laguerre-Hahn regular functional and $\{\phi_n\}, \{\Omega_n\}$ and $\{Q_n\}$ be the sequences of the corresponding orthogonal polynomials, associated polynomials of the second kind and functions of the second kind, respectively. In this talk we give two characterizations of Laguerre-Hahn orthogonal polynomials:
a) in terms of a first order structure relation for $\{\phi_n\}, \{\Omega_n\}$ and $\{Q_n\}$;
b) in terms of a second order differential equation for the vectors $[\phi_n,\ \Omega_n]^T$ and for $\{Q_n\}$.

18:00-18:30 Rafael Hernández Heredero, Polytechnical University of Madrid
The complex Toda lattice: The Bäcklund transformation and the structure of the space of solutions

Using techniques from Orthogonal Polynomial theory and Operator theory, we give a sufficient condition for the existence of the Bäcklund transformation of the Toda lattice, a well known integrable system. Using this transformation we sketch the structure of the space of solutions of the system.