PhD Program in Mathematics UC|UP

Partial Differential Equations

Spring 2008; TUE 11:00–13:00 and 14:30–16:30

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COURSE DESCRIPTION

The course is an introduction to the study of partial differential equations (PDEs) using functional analysis and energy methods. Questions of existence, uniqueness and regularity for weak solutions to linear elliptic and parabolic PDEs will be emphasized. Various nonlinear PDEs will also be studied, using a variety of different approaches, like variational and monotonicity methods, fixed-point theorems or intrinsic scaling.

TEXT BOOKS

I will extensively follow the celebrated book of L.C. Evans [8], with complements and extensions from a variety of sources (listed in the references), mainly [1, 6, 14, 18]. For the last chapter, I will use my book [20] (and also [7] and [17]).

HOMEWORK

There will be two **homework sets**. They will be available a week in advance, and due in class on the dates specified. No late homework will be accepted.

Homework 1: due on February 26, by 11:00.

Homework 2: due on March 25, by 11:00.

EXAMS

There will be three one-hour **in-class exams** (on March 4, April 8 and April 29). The lowest in-class exam grade will be dropped. There will also be a three-hour **final exam** on May 20.

GRADING

Homework sets: **0.05** each; In-class exams: **0.15** each; Final exam: **0.6**.

SYLLABUS

1. SECOND ORDER LINEAR ELLIPTIC EQUATIONS

- Existence of weak solutions: Lax–Milgram theorem; energy estimates; Fredholm alternative.
- Regularity in the interior and up to the boundary: difference quotient method of Nirenberg.
- Maximum principles. Harnack inequality.
- De Giorgi–Nash–Moser theory: local boundedness and Hölder continuity.

2. SECOND ORDER LINEAR PARABOLIC EQUATIONS

- Existence: Galerkin method.
- Regularity theory and maximum principles.

3. LINEAR SEMIGROUP THEORY

- Generators and resolvents.
- Hille-Yosida theorem.
- Application to second order parabolic PDEs.

4. THE CALCULUS OF VARIATIONS

- Euler–Lagrange equation.
- Existence of minimizers: coercivity, lower semi-continuity and convexity. Weak solutions of the Euler–Lagrange equation.
- Regularity. Unilateral constraints: variational inequalities; free boundary problems.

5. NONVARIATIONAL TECHNIQUES

- Monotonicity methods: monotone operators; Minty–Browder lemma.
- Fixed point methods: Banach and Schauder fixed point theorems.

6. DEGENERATE AND SINGULAR PDEs

- The p–Laplace equation: Dirichlet problem and weak solutions; regularity theory.
- The parabolic case: regularity through intrinsic scaling.
- The infinity Laplacian.

References

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[4] E. DiBenedetto, *Degenerate Parabolic Equations*, Springer, 1993.

[5] E. DiBenedetto, *Real Analysis*, Birkhäuser, 2002.

[6] E. DiBenedetto, *Partial Differential Equations*, 2nd ed., Birkhäuser, 2008.

[7] E. DiBenedetto, J.M. Urbano and V. Vespri, *Current issues on singular and degenerate evolution equations*, in: Handbook of Differential Equations, Evolutionary Equations, Vol. 1, pp. 169-286, Elsevier, 2004.

[8] L.C. Evans, *Partial Differential Equations*, Graduate Studies in Mathematics, Vol. **19**, American Mathematical Society, 1998.

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[10] D. Gilbarg and N. Trudinger, *Elliptic Partial Differential Equations of Second Order*, 2nd ed., Springer, 1983.

[11] E. Giusti, *Metodi Diretti nel Calcolo delle Variazioni*, Unione Matematica Italiana, 1994.

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[14] D. Kinderlehrer and G. Stampacchia, *An Introduction to Variational Inequalities and Their Applications*, Academic Press, 1980.

[15] O. Ladyzhenskaya, V. Solonnikov and N. Ural'tseva, *Linear and Quasilinear Equations of Parabolic Type*, American Mathematical Society, 1968.

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[19] M. Taylor, *Partial Differential Equations*, Vols. I–III, Springer, 1996.

[20] J.M. Urbano, *The Method of Intrinsic Scaling*, Lecture Notes in Mathematics, Vol. **1930**, Springer, 2008.

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