PhD Program in Mathematics UC|UP

Partial Differential Equations

Spring 2016; MON, 10:30–15:30

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COURSE DESCRIPTION

The course is an introduction to the study of partial differential equations (PDEs) using functional analysis and energy methods. Questions of existence, uniqueness and regularity for weak solutions to linear elliptic and parabolic PDEs will be emphasized. Various nonlinear PDEs will also be studied, using a variety of different approaches, like variational and monotonicity methods, fixed-point theorems or intrinsic scaling.

TEXT BOOKS

I will extensively follow the celebrated book of L.C. Evans [7], with complements and extensions from a variety of sources (listed in the bibliography), mainly [1, 5, 12, 16]. For the last chapter, I will use my book [17] and the set of notes [18].

CLASS WORK

Sets of exercises, mainly taken from Evans' book, will be proposed to the students and corrected in class.

EVALUATION

There will be a two-hour **intermediate exam** (on April 4) and a three-hour **final exam** (on May 30).

Additionally, each student will make a short presentation of a topic of his/her choosing during the semester.

GRADING

Oral presentation: **0.25**; Intermediate exam: **0.25**; Final exam: **0.5**.

SYLLABUS

0. CRASH COURSE ON SOBOLEV SPACES

1. SECOND ORDER LINEAR ELLIPTIC EQUATIONS

- Existence of weak solutions: Lax–Milgram theorem; energy estimates; Fredholm alternative.
- Regularity in the interior and up to the boundary: difference quotient method of Nirenberg.
- Maximum principles. Harnack inequality.
- De Giorgi–Nash–Moser theory: local boundedness and Hölder continuity.

2. SECOND ORDER LINEAR PARABOLIC EQUATIONS

- Existence: Galerkin method.
- Regularity theory and maximum principles.

3. THE CALCULUS OF VARIATIONS

- Euler–Lagrange equation.
- Existence of minimizers: coercivity, lower semi-continuity and convexity. Weak solutions of the Euler–Lagrange equation.
- Regularity. Unilateral constraints: variational inequalities; free boundary problems.

4. NONVARIATIONAL TECHNIQUES

- Monotonicity methods: monotone operators; Minty–Browder lemma.
- Fixed point methods: Banach and Schauder fixed point theorems.

5. DEGENERATE AND SINGULAR PDEs

- The p–Laplace equation: Dirichlet problem and weak solutions; regularity theory.
- The parabolic case: regularity through intrinsic scaling.
- The infinity Laplacian.

Bibliography

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[4] E. DiBenedetto, *Real Analysis*, Birkhäuser, 2002.

[5] E. DiBenedetto, *Partial Differential Equations*, 2nd ed., Birkhäuser, 2008.

[6] E. DiBenedetto, J.M. Urbano and V. Vespri, *Current issues on singular and degenerate evolution equations*, in: Handbook of Differential Equations, Evolutionary Equations, Vol. 1, pp. 169-286, Elsevier, 2004.

[7] L.C. Evans, *Partial Differential Equations: Second Edition*, Graduate Studies in Mathematics, Vol. **19**, American Mathematical Society, 2010.

[8] M. Giaquinta, *Introduction to Regularity Theory for Nonlinear Elliptic Systems*, Birkhäuser, 1993.

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[11] Q. Han and F. Lin, *Elliptic Partial Differential Equations*, Courant Lecture Notes in Mathematics, Vol. **1**, American Mathematical Society, 1997.

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[13] O. Ladyzhenskaya, V. Solonnikov and N. Ural'tseva, *Linear and Quasilinear Equations of Parabolic Type*, American Mathematical Society, 1968.

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[15] P. Lindqvist, *Notes on the p-Laplace equation*, preprint, University of Jyväskylä, 2005.

[16] J.L. Lions, *Quelques Méthodes de Résolution des Problèmes aux Limites non Linéaires*, Dunod, 1969.

[17] J.M. Urbano, *The Method of Intrinsic Scaling*, Lecture Notes in Mathematics, Vol. **1930**, Springer, 2008.

[18] J.M. Urbano, *An introduction to the infinity-Laplacian*, Coimbra 2013. [http://www.mat.uc.pt/∼jmurb/notes_final.pdf]