

PhD Program in Mathematics UC|UP

# Partial Differential Equations

Spring 2017; TUE, 16:30–18:30 and THU, 14:30–16:30

JOSÉ MIGUEL URBANO  
[www.mat.uc.pt/~jmurb](http://www.mat.uc.pt/~jmurb)

## COURSE DESCRIPTION

The course is an introduction to the study of elliptic partial differential equations (PDEs) using functional analysis and energy methods, with an emphasis on regularity issues. An important part of the course is devoted to nonlinear equations.

## TEXT BOOKS

I will use either the celebrated book of L.C. Evans [3], or my sets of notes [9] and [10].

## CLASS WORK

Sets of exercises, mainly taken from Evans' book, will be proposed to the students and corrected in class.

## EVALUATION

There will be a two-hour **intermediate exam** (on April 6) and a three-hour **final exam** (on May 25).

Additionally, each student will make a short presentation of a topic of his/her choosing during the semester.

## GRADING

Oral presentation: **0.25**; Intermediate exam: **0.25**; Final exam: **0.5**.

# SYLLABUS

## 0. CRASH COURSE ON SOBOLEV SPACES

### 1. SECOND ORDER LINEAR ELLIPTIC EQUATIONS

- Existence of weak solutions; energy methods.
- Smooth coefficients: difference quotients and interior regularity.

### 2. DE GIORGI-NASH-MOSER THEORY

- Hilbert's 19th problem.
- Local boundedness and Hölder continuity.
- Moser's Harnack inequality.

### 3. THE CALCULUS OF VARIATIONS

- Euler–Lagrange equation.
- Existence of minimizers: coercivity, lower semi-continuity and convexity. Weak solutions of the Euler–Lagrange equation.
- Regularity. Unilateral constraints: variational inequalities; free boundary problems.

### 4. AN INTRODUCTION TO THE $\infty$ -LAPLACIAN

- The Lipschitz extension problem.
- Comparison with cones.
- Viscosity solutions of  $\Delta_\infty = 0$ .
- Harnack inequality and locally Lipschitz regularity.
- Uniqueness: the proof of Armstrong and Smart.

## Bibliography

- [1] H. Brézis, *Analyse Fonctionnelle*, Masson, 1983.
- [2] H. Brézis and F. Browder, *Partial differential equations in the 20th century*, Advances in Mathematics **135** (1998), 76-144.
- [3] L.C. Evans, *Partial Differential Equations: Second Edition*, Graduate Studies in Mathematics, Vol. **19**, American Mathematical Society, 2010.
- [4] M. Giaquinta, *Introduction to Regularity Theory for Nonlinear Elliptic Systems*, Birkhäuser, 1993.
- [5] D. Gilbarg and N. Trudinger, *Elliptic Partial Differential Equations of Second Order*, 2nd ed., Springer, 1983.
- [6] E. Giusti, *Metodi Diretti nel Calcolo delle Variazioni*, Unione Matematica Italiana, 1994.
- [7] Q. Han and F. Lin, *Elliptic Partial Differential Equations*, Courant Lecture Notes in Mathematics, Vol. **1**, American Mathematical Society, 1997.
- [8] D. Kinderlehrer and G. Stampacchia, *An Introduction to Variational Inequalities and Their Applications*, Academic Press, 1980.
- [9] J.M. Urbano, *A very short introduction to regularity theory for linear elliptic equations*, Universidade de Coimbra, 2016.
- [10] J.M. Urbano, *An introduction to the  $\infty$ -Laplacian*, Universidade de Coimbra, 2017.