PhD Program in Mathematics UC|UP

Partial Differential Equations

Spring 2008

HOMEWORK 2

1. Assume U is connected. Use the maximum principle to show that any smooth solution of the Neumann boundary-value problem

$$\begin{cases} \Delta u = 0 & \text{in } U \\ \dfrac{\partial u}{\partial \nu} = 0 & \text{on } \partial U \end{cases}$$

is constant.

2. Assume

$$\psi(k_m, r_m) \le C \, \frac{2^{m+1+\frac{2m}{n}}}{k^{\frac{2}{n}}} \, \psi(k_{m-1}, r_{m-1})^{1+\frac{2}{n}}, \qquad m = 0, 1, 2, \ldots$$

Prove by induction that, choosing k sufficiently large,

$$\psi(k_m, r_m) \le \frac{\psi(k_0, r_0)}{\gamma^m} \ , \ \forall\, m = 0, 1, 2, \ldots$$

for some constant $\gamma > 1$.

3. Reproduce, from the original paper of J. Moser concerning Harnack's theorem for elliptic PDEs, the proof that the interior Hölder continuity of weak solutions is a consequence of Harnack's inequality.

Be prepared to explain the reasoning on the blackboard.

Due on March 25, 2008