Large solutions for a class of nonlinear elliptic equations with gradient terms

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In the first part of the seminar we present an existence (and regularity) result for a class of nonlinear elliptic equations whose model is

$$A(u) + g(x,u)|\nabla u|^q + h(u) = f(x) \quad \text{in } \Omega$$

(A(u) a Leray-Lions operator, $1 < q \leq 2$, $g(x,s)$ "coercive", $f \in L^1(\Omega)$ and h unbounded) equipped with the condition $u = +\infty$ on $\partial\Omega$. Solutions that satisfy (in some sense) such boundary condition are known as "large solutions". In the second part we present some qualitative properties for the solution of the "model problem"

$$\begin{cases} -\Delta u + |\nabla u|^q + u = f(x) & \text{in } \Omega \\ u = +\infty & \text{on } \partial\Omega\,, \end{cases}$$

with f smooth. The interest in such problem comes from a related stochastic control problem with state constraint.