ON WEIGHTED ESTIMATES FOR THE AVERAGING INTEGRAL OPERATOR

BOHUMÍR OPIC

ABSTRACT. Let $1 < p < +\infty$ and let v be a non-decreasing weight on the interval $(0, +\infty)$. We prove that if the averaging operator $(Af)(x) := \frac{1}{x}\int_0^x f(t)\,\mathrm{d}t$, $x \in (0, +\infty)$, is bounded on the weighted Lebesgue space $L^p((0, +\infty); v)$, then there exist $\varepsilon_0 \in (0, p-1)$ such that the operator A is also bounded on the space $L^{p-\varepsilon}((0, +\infty); v(x)^{1+\delta}\, x^{\gamma})$ for all ε, δ, $\gamma \in [0, \varepsilon_0)$. Conversely, assuming that the operator A is bounded on the space $L^{p-\varepsilon}((0, +\infty); v(x)^{1+\delta}\, x^{\gamma})$ for some $\varepsilon \in [0, p-1)$, $\delta \geq 0$ and $\gamma \geq 0$, we prove that the operator A is bounded on the space $L^p((0, +\infty); v)$. Results have been obtained in collaboration with my colleague Jiří Rákosník.

MATHEMATICAL INSTITUTE AS CR, ŽITNÁ 25, 11567 PRAHA 1, CZECH REPUBLIC
E-mail address: `opic@math.cas.cz`