Risk and Decision Making (Part II - Game Theory) Mock Exam
MIT/Portugal 2 pages
Professor João Soares 2007/08

1. Consider the game defined by the Kuhn tree of Figure 1.

   (a) Describe the game in plain english and find its equivalent strategic form.

   (b) Should each Player use their maxmin (mixed) strategy, what is the expected outcome of the game?

   (c) Find all pure strategic equilibria of this game.

   (d) Should Player I allow Player II to make its move what is the natural outcome of this game?

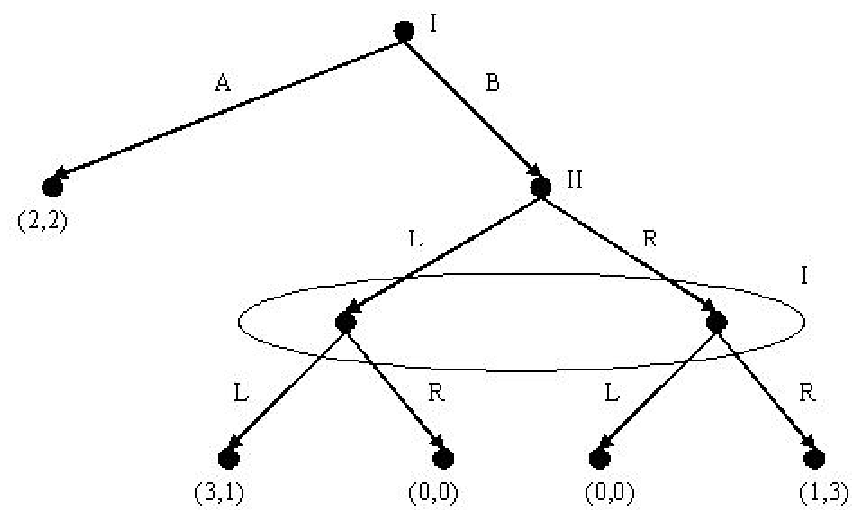

Figure 1:

2. Army A has a single plane with which it can attack one of three possible targets $1, 2,$ and 3. Army D has one anti-aircraft gun that can be assigned to defend one of these three targets. Army A can destroy a target only if the target is undefended and A attacks it. The value of destroying target k is k for army A. Similarly, the value of defending an attack on target k is k for army D. Both armies take their actions simultaneously, so

the situation is summarized by the following normal form game:

$$\begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} \overset{\begin{array}{ccc} D_1 & D_2 & D_3 \end{array}}{\begin{bmatrix} (0,1) & (1,0) & (1,0) \\ (2,0) & (0,2) & (2,0) \\ (3,0) & (3,0) & (0,3) \end{bmatrix}}.$$

Find as many strategic equilibria as you can, including the mixed one (Suggestion: eliminate one of the pure strategies of each player by domination).

3. Consider the cooperative TU bimatrix game:

$$\begin{bmatrix} (3,2) & (4,1) & (4,2) \\ (4,2) & (2,3) & (4,1) \\ (1,3) & (3,0) & (4,3) \end{bmatrix}.$$

   (a) Find the TU-values.

   (b) Find the associated side payment.

   (c) Find the optimal threat strategies.

4. (a) Define what it means for a vector $(\bar{u}, \bar{v}) \in S$, where S is the NTU-feasible set, to be Pareto optimal in a two-player NTU game.

   (b) Consider the cooperative NTU bimatrix game:

$$\begin{bmatrix} (2,4) & (6,0) \\ (9,1) & (3,4) \end{bmatrix}.$$

   Let $(u^*, v^*) = (1, 0)$ be the disagreement point (or threat point, or status-quo point). Find the NTU-value (*i.e.*, the Nash bargaining solution).

5. Consider the three-person game in coalitional form with characteristic function,

$$\begin{array}{llll} & \bar{v}(\{1\}) = 0 & \bar{v}(\{1,2\}) = 2 & \\ \bar{v}(\emptyset) = 0 & \bar{v}(\{2\}) = 1 & \bar{v}(\{1,3\}) = 3 & \bar{v}(\{1,2,3\}) = 10 \\ & \bar{v}(\{3\}) = 2 & \bar{v}(\{2,3\}) = 6 & \end{array}$$

   (a) How would you find the least rational core? (establish the linear program)

   (b) How would you find the nucleolus? Be succinct.