

On the evolution of systems defined by computable Initial Value Problems

In this talk we will investigate some computability/non-computability issues regarding Initial Value Problems (IVP) defined with ordinary differential equations (ODEs).

More concretely, we explore the following question: given the data defining an ODE plus a set of initial conditions, compute, if possible, the maximal interval of time where the solution is defined, and obtain an algorithm that gives the corresponding solution over this time interval. This is not achieved by standard procedures because, in general, no Lipschitz condition is satisfied over this whole maximal interval.

Using the framework of computable analysis, we will show that if the data defining the IVP is computable then: (a) the maximal interval where solution is defined is recursively enumerable, but in general not recursive; (b) the solution is computable over this interval and (c) the problem of deciding if this maximal interval is finite or not is undecidable.

Joint work with N. Zhong