GILDA FERREIRA, *An Embedding of the Predicate Calculus.*
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In the context of *Proof Theory*, with the purpose of treating proofs as formal objects in order to study them using mathematical techniques, some *formal deductive systems* were introduced and developed. Among such systems is the one known as *Natural Deduction*, proposed by Gerhard Gentzen in 1935, intended to be as close as possible to the way people actually reason. The Natural Deduction system is settled on a deep symmetry conferred by the usual *introduction* and *elimination* rules (inverse in a certain sense). In [1], Dag Prawitz contributed to the development of the system with his *normalization* result, closely connected, in the context of Sequent Calculus, to the Cut Elimination Theorem.

According to Jean-Yves Girard et al. [2], Natural Deduction 'is only satisfactory for $(\wedge, \rightarrow, \forall)$ fragment of the language', and they add that 'for [the full] fragment, our syntactic methods are frankly inadequate'.

Recently, in [3], Fernando Ferreira suggests a way of avoiding the full fragment, presenting an embedding of the propositional calculus into a second-order predicative calculus based in $\rightarrow$ and $\forall$. Such an embedding is possible since $\neg$, $\wedge$ and $\vee$ can be defined in terms of $\rightarrow$ and $\forall$ — e.g. in the disjunction case $A \vee B =_{def} \forall F((A \rightarrow F) \rightarrow ((B \rightarrow F) \rightarrow F))$, with F a new propositional variable — and because it is possible to extend predicative instantiations of F to *any* formula of the language.

In this talk, we extend the embedding presented in [3] (in the context of propositional calculus) to the predicate calculus. As an application we show the *existence property* of intuitionistic predicate logic.

[1] D. Prawitz. *Natural Deduction.* Almkvist & Wiksell, Stockholm, 1965.

[2] J.-Y. Girard, Y. Lafont and P. Taylor. *Proofs and Types.* Cambridge University Press, 1989.

[3] F. Ferreira. *Comments on Predicative Logic.* To appear in *Journal of Philosophical Logic.* Available in http://alf1.cii.fc.ul.pt/~ferferr/comments.pdf