Some Complexity Results on Cut Elimination in First Order Based Logics

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Abstract

The worst-case complexity of cut elimination in sequent calculi for first order based logics is investigated in terms of the increase in logical depth of the deduction. It is shown that given a calculus satisfying a general collection of sufficient conditions for cut elimination and given a deduction with cuts, there exists a cut free deduction with a logical depth, in the worst case, hyper-exponentially greater than the logical depth of the original deduction. Moreover an interesting relation, as far as we know not yet reported in the literature, between this complexity and the greatest cut length of a pair of introduction rules in the calculus for a constructor, is established. By the cut length of a pair of introduction rules of a constructor it is meant the minimal length of a cut sequence for that pair, where a cut sequence for a pair of introduction rules is a sequence formed by premises of that rules, without contexts, that when combined in a deduction using a generic cut rule, respecting the sequence order, lead to the empty sequent.

The non-elementary bound in the worst case complexity of the cut elimination procedure was expected since it is a lower bound of cut elimination in sequent calculi for classical first order logic [3] and this logic satisfies the sufficient conditions for cut elimination established herein. The present study on the complexity of cut elimination accompanies the interest that exists in the literature in the study of the length of proofs, in part motivated by its relationship with computational complexity [2, 4, 1].

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