

# Issues in a calculus of multiary sequent terms

J. Espírito Santo\*, M. J. Frade<sup>+</sup>, L. Pinto\*

\*Departamento de Matemática    +Departamento de Informática  
Universidade do Minho            Universidade do Minho

In this talk we overview our study on an extension of the  $\lambda$ -calculus introduced in [1], exhibiting the features of *multiarity* and *generality*. The former feature is the ability of applying a term to a list of arguments. The latter is the ability of specifying a future use, or “continuation”, for a (possibly multiary) application. The calculus was named the generalised multiary  $\lambda$ -calculus, or the  $\lambda\mathbf{J}^m$ -calculus for short. In its simply typed version, the calculus corresponds to a sequent calculus, and the novelty of the mentioned features relates to the novelty of the left introduction rule relatively to a natural deduction format.

The calculus was introduced in [1] as a calculus of multiary sequent terms (in the sense of [4]) for a study of permutative conversions in sequent calculus. The main lesson of this study is the existence of a relationship between permutative conversions, subsystems of  $\lambda\mathbf{J}^m$  and the features of generality and multiarity. Moreover, some subsystems turn out to be isomorphic to fragments of natural deduction, taking natural deduction in the extended sense of von Plato [5].

Later work [2] observes an overlap between the features of generality and multiarity. In [2] the overlap is mainly used to transfer results about reduction from  $\Lambda J$  to  $\lambda\mathbf{J}^m$ , where  $\Lambda J$  is the  $\lambda$ -calculus with generalised application introduced in [3]. In addition this overlap suggests a refinement of the original view of  $\lambda\mathbf{J}^m$  as obtained from  $\lambda$  by modularly adding the two new features. This refinement is ongoing work.

Another issue we are considering is that of obtaining a  $\lambda$ -term in  $\beta$ -normal form out of a  $\lambda\mathbf{J}^m$ -term. This requires the combination of reduction and permutative conversion and, in particular, raises questions of termination. These questions relate to the problem of preservation of strong normalisation in calculi of explicit substitutions.

## References

- [1] J. Espírito Santo and L. Pinto, *Permutative conversions in intuitionistic multiary sequent calculus with cuts*, in M. Hoffman (ed.), Proc. of TLCA 2003, Lecture Notes in Computer Science, vol. 2701, 286–300, Springer-Verlag, 2003.
- [2] J. Espírito Santo and L. Pinto, *Confluence and strong normalisation of the generalised multiary  $\lambda$ -calculus*, in Stefano Berardi, Mario Coppo, Ferruccio Damiani (eds.), Revised selected papers from the International Workshop TYPES 2003, Lecture Notes in Computer Science, vol. 3085, Springer-Verlag, 2004.
- [3] F. Joachimski and R. Matthes, *Standardization and confluence for a lambda calculus with generalized applications*, in Leo Bachmair (ed.), Proceedings of 11th RTA 2000 (Norwich), Lecture Notes in Computer Science, vol. 1833, pp. 141-155, Springer-Verlag, 2000.
- [4] H. Schwichtenberg, *Termination of permutative conversions in intuitionistic Gentzen calculi*, *Theoretical Computer Science*, **212** (1999) 247–260.
- [5] J. von Plato, *Natural deduction with general elimination rules*, *Annals of Mathematical Logic*, 40 (2001) 541–567.