PROOF SEARCH AND COUNTERMODEL CONSTRUCTION IN BI-INTUITIONISTIC PROPOSITIONAL LOGIC

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Bi-intuitionistic propositional logic (also known as Heyting-Brouwer logic, subtractive logic) extends intuitionistic propositional logic with a connective that is dual to implication, called coimplication. It is the logic of bi-[Cartesian closed] categories and it also has a Kripke semantics. It first got the attention of C. Rauszer who studied it in a number of papers. Later, it has been investigated in particular by T. Crolard. There are conjectured applications in computer science related to the duality of values and continuations (P.-L. Curien and H. Herbelin, P. Wadler), even if the categorical models are disappointingly poor because of Joyal's lemma.

From the point of view of structural proof theory, bi-intuitionistic logic is quite interesting. There is a simple sequent calculus that dualizes Dragalin's sequent calculus for intuitionistic logic (no side succedent formulae in the implication right rule and no side antecedent formulae in the coimplication left rule). But this sequent calculus does not enjoy cut elimination, although several authors have mistakenly asserted or proved it. The failure is very similar to that in the case of System S5.

We present an alternative sequent calculus with explicit possible worlds designed according to the methodology of S. Negri for designing sequent calculi for modal logics. That sequent calculus has cut elimination. Moreover, it also admits an algorithmic refinement that describes a proof search/countermodel construction procedure with a non-trivial termination argument.

Our sequent calculus bears similarities to T. Crolard's sequent calculus with dependency tracking. In a sense, it is its rational reconstruction from Kripke semantics.

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